

SUBJECT : MATHEMATICS
CLASS : XII
BLUE PRINT

Sl. No.	Form of question		V.S.A.	S.A.	L.A.	Unit wise weight age
	Chapters					
1.	Relations & functions		1(1)	1(4)	-	10
2.	Inverse Trigonometric functions		1(1)	1(4)	-	
3.	Matrices		3(1)	1(4)	1(6)	13
4.	Determinants					
5.	Continuity and differentiability		1(1)	2(4)	-	44
6.	Application of Derivatives		-	-	1(6)	
7.	Integrals		1(1)	2(4)	-	
8.	Applications of Integrals		-	-	-1(6)	
9.	Differential Equations		-	2(4)	1(6)	
10.	Vectors		2(1)	1(4)		17
11.	Three dimensional Geometry		1(1)	1(4)	1(6)	
12.	Linear Programming		-	-	1(6)	06
13.	Probability		-	1(4)	1(6)	10
Total			10(1)	12(4)	7(6)	100
No. of Questions		10 questions of 1 mark each = 10 marks 12 questions of 4 marks each = 48 marks 07 questions of 6 marks each = 42 marks				
		29 questions		100 marks		

MODEL TEST PAPER-1

Time Allowed : 3 hrs.

Max. Marks : 100

General Instructions :

- (i) All questions are compulsory
- (ii) Question numbers 01 to 10 (Part-A) are of 1 mark each.
- (iii) Question numbers 11 to 22 (Part-B) are of 4 marks each.
- (iv) Question numbers 23 to 29 (Part-C) are of 6 marks each.

PART-A

1. If $f : R \rightarrow R$ be given by $f(x) = (3 - x^3)^{1/3}$, then $f(x)$ is –
(a) $x^{1/3}$ (b) x^3 (c) x (d) $(3 - x^3)$
2. If $\tan^{-1} x = y$, then –
(a) $0 \leq y \leq \pi$ (b) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (c) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (d) $0 < y < \pi$
3. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then
(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta\gamma = 0$ (c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$
4. Find $\frac{d^2y}{dx^2}$ if $y = \log x$
5. The rate of change of the area of a circle with respect to its radius r at $r = 6\text{cm}$ is –
(a) 10π (b) 12π (c) 8π (d) 11π
6. Solve $\frac{dy}{dx} = 1 + x + y + xy$
7. Evaluate $\int \sqrt{1 + \sin 2x} dx$
8. If \vec{a} is a non-zero vector of magnitude 'a' and 'l' a non-zero scalar, then $\lambda \vec{a}$ is a unit vector if –
(a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$
9. Distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is –
(a) $\frac{2}{\sqrt{29}}$ units (b) $\frac{4}{\sqrt{29}}$ (c) 2 units (d) 4 units

10. The planes $2x - y + 4z = 5$ and $5x - \frac{5}{2}y + 10z = 6$ are –
- (a) Perpendicular (b) Parallel
- (c) Intersect along y-axis (d) Pass through $(0; 0, \frac{5}{4})$

PART-B

11. Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$
12. Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$ where –
 $y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$ show that f is invertible. Find the inverse.
13. Solve the differential equation – $\frac{dy}{dx} + y \cot x = 2 \cos x$

OR

Find the differential equation of all circles which pass through origin and whose centre lies on y-axis.

14. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$ find $\frac{d^2y}{dx^2}$
15. Find the value of k so that the following function f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \pi/2 \\ 3 & \text{if } x = \pi/2 \end{cases} \quad \text{at } x = \pi/2$$

16. Use differential to approximate $(26)^{1/3}$

OR

Find the intervals in which function $f(x)$ is increasing or decreasing

$$f(x) = 2x^3 - 15x^2 + 36x + 17$$

17. Evaluate $\int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin x}$
18. Evaluate $\int x \tan^{-1} x dx$ or $\int \frac{x}{\sqrt{8+x-x^2}} dx$
19. Evaluate $\int_{-1}^2 (7x-5) dx$ as a limit of sum **OR** Evaluate $\int \frac{dx}{x^4 + 1}$

20. If $\vec{a}, \vec{b}, \vec{c}$ be the vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

21. Find the shortest distance between two lines whose equations are given below –

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\vec{r} = (S+1)\hat{i} + (2S-1)\hat{j} - (2S+1)\hat{k}$$

22. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

PART-C

23. Obtain the inverse of the following matrix using elementary operations

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}, \quad \text{OR}$$

Using properties of determinants show that –

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1 \right) = abc + ab + bc + ca$$

24. Solve the following system of linear equations by matrix method.

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

25. A point on the hypotenuse of a triangle is at distance a and b from the sides of the

triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}} \right)^{\frac{3}{2}}$

26. Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

27. Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also find the length of the perpendicular.

28. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per killogram, find the minimum cost of producing this new cereal if bran costs Rs.5 per kilogram and rice costs Rs.4 per kilogram.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

SOLUTION SET-I

Part-A

1. (c) x
2. (c) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
3. (c) $1 - \alpha^2 - \beta\gamma = 0$
4. Find $\frac{d^2y}{dx^2}$ if $y = \log x$

$$\frac{dy}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

5. (b) 12P
6. Solve $\frac{dy}{dx} = 1 + x + y + xy$

$$\frac{dy}{dx} = 1 + x + y + xy = (1+x) + y(1+x)$$

$$\text{or } \frac{dy}{1+y} = (1+x) dx$$

Integrating both sides,

$$\log(1+y) = x + \frac{x^2}{2} + c$$

7. Evaluate $\int \sqrt{1 + \sin 2x} dx$

Solution :

$$\begin{aligned} & \int \sqrt{1 + \sin 2x} dx \\ &= \int \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} dx \\ &= \int \sqrt{(\sin x + \cos x)^2} dx \\ &= \int (\sin x + \cos x) dx \end{aligned}$$

$$= \int \sin x dx + \int \cos x dx$$

$$= -\cos x + \sin x + c$$

8. (d) $a = \frac{1}{|\lambda|}$

9. (a) $\frac{2}{\sqrt{29}}$ units (b) Parallel

PART-B

11. Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$

Solution : Let $\sin^{-1} \frac{12}{13} = x$
 $\cos^{-1} \frac{4}{5} = y$
 $\tan^{-1} \frac{63}{16} = z$

Then $\sin x = \frac{12}{13}$
 $\cos y = \frac{4}{5}$
 $\tan z = \frac{63}{16}$

Therefore $\sin x = \frac{5}{13}$
 $\cos y = \frac{3}{5}$
 $\tan x = \frac{12}{5}$
 $\tan y = \frac{3}{4}$

We have, $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$$\begin{aligned}
&= \frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{12}{5} \times \frac{3}{4}} \\
&= -\frac{63}{16}
\end{aligned}$$

Hence,

$$\tan(x+y) = -\tan z$$

$$\text{i.e. } \tan(x+y) = \tan(-z)$$

$$\text{or } \tan(x+y) = \tan(\pi - z)$$

$$\text{therefore, } x+y = -z$$

$$\text{or } x+y = \pi - z$$

Since x, y, z are positive, therefore $x+y \neq -z$.

$$\text{Hence } x+y+z = \pi$$

$$\text{or } \sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$$

hence shown

12. Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x+3$, where

$y = \{y \in N : y = 4x+3 \text{ for some } x \in N\}$. Show that f is invertible. Find the inverse.

Solution :

A function is invertible if its is a bijection for any $x, y \in N$, we find that

$$f(x) = f(y)$$

$$\Rightarrow 4x+3 = 4y+3$$

$$\Rightarrow x = y$$

So, $f : N \rightarrow Y$ is one-one

Let y be an arbitrary element of Y . Then, there exists $x \in N$ such that

$$y = 4x+3 \quad \text{or} \quad y = f(x)$$

Thus, for each $y \in N$ there exists $x \in N$ such that $f(x) = y$

So, $f : N \rightarrow Y$ is onto

Thus, $f : N \rightarrow Y$ is both one-one and onto. Consequently, it is invertible. Hence shown

$$\text{Now, } f \circ f^{-1}(x) = x \quad \forall \text{ all } x \in y$$

$$\text{or } f(f^{-1}(x)) = x \quad \forall \text{ all } x \in y$$

or $4f^{-1}(x)+3=x \quad \forall \text{ all } x \in y$

or $f^{-1}(x)=\frac{x-3}{4} \quad \forall \text{ all } x \in y$

Hence $f^{-1} : Y \rightarrow N$ is given by

$$f^{-1}(x)=\frac{x-3}{4} \quad \forall \text{ all } x \in y$$

13. Solve $\frac{dy}{dx} + y \cot x = 2 \cos x$

Solution :

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

Where, $P = \cot x$ and $Q = 2 \cos x$

\therefore Integrating factor (I.F.)

$$\begin{aligned} &= e^{\int P dx} \\ &= e^{\int \cot x dx} \\ &= e^{\log|\sin x|} \\ &= \sin x \end{aligned}$$

Multiply both sides of the differential equation by –

I.F. = $\sin x$, and integrating, we get–

$$y(\sin x) = \int (2 \cos x \sin x) dx + C \quad \left[\text{Using } y(\text{I.F.}) = \int Q(\text{I.F.}) dx + C \right]$$

or $y(\sin x) = \int \sin 2x dx + C$

or $y(\sin x) = -\frac{\cos 2x}{2} + C$, Which is the required solution.

OR

Find the differential equation of all circles which pass through the origin and whose centre lies on y-axis.

Solution :

The equation of the family of circles touching x-axis at the origin is

$$(x-0)^2 + (y-a)^2 = a^2$$

or, $x^2 + y^2 - 2ay = 0$ ----- (i)

Where a is a parameter. This equation contains only one arbitrary constant, we differentiate it once w.r.t. x , so that–

$$2x + 2y \frac{dy}{dx} - 2a \frac{dy}{dx} = 0$$

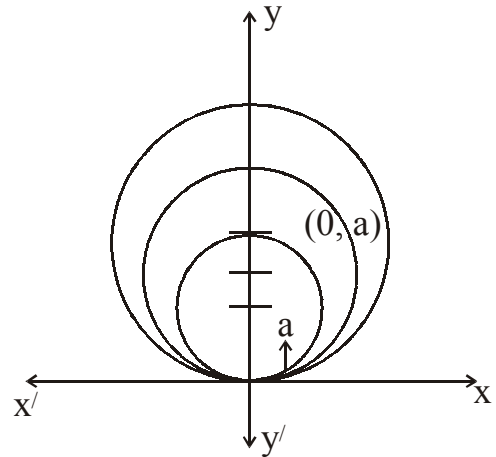
$$\text{or } a = \frac{x + y \left(\frac{dy}{dx} \right)}{\left(\frac{dy}{dx} \right)} \text{----- (ii)}$$

Putting the value of a from (ii) in (i) we get –

$$x^2 + y^2 = 2y \left(\frac{x + y \left(\frac{dy}{dx} \right)}{\frac{dy}{dx}} \right)$$

$$\text{or } (x^2 - y^2) \frac{dy}{dx} = 2xy$$

This is the required differential equation.



14. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2 y}{dx^2}$

Solution :

$$x = a(\cos t + t \sin t)$$

$$\begin{aligned} \therefore \frac{dx}{dt} &= \frac{d}{dt} \{a(\cos t + t \sin t)\} \\ &= a \left\{ \frac{d}{dt}(\cos t) + \frac{d}{dt}(t \sin t) \right\} \\ &= a \left\{ -\sin t + t \frac{d}{dt}(\sin t) + \sin t \frac{d}{dt}(t) \right\} \\ &= a \{-\sin t + t \cos t + \sin t\} \\ &= at \cdot \cos t \text{----- (i)} \end{aligned}$$

$$y = a(\sin t - t \cos t)$$

$$\begin{aligned} \therefore \frac{dy}{dt} &= \frac{d}{dt} \{a(\sin t - t \cos t)\} \\ &= a \left\{ \frac{d}{dt}(\sin t) - \frac{d}{dt}(t \cos t) \right\} \\ &= a \left(\cos t - t \frac{d}{dt}(\cos t) - \cos t \frac{d}{dt}(t) \right) \end{aligned}$$

$$= a(\cos t + t \sin t - \cos t)$$

$$= at \cdot \sin t \text{ ----- (ii)}$$

Dividing equation (ii) by (i) we get –

$$\frac{dy}{dx} = \frac{at \sin t}{at \cos t}$$

$$= \tan t$$

Differentiating both sides w.r.t. x we get –

$$\frac{d^2y}{dx^2} = \frac{d}{dt}(\tan t) \cdot \frac{dt}{dx}$$

$$= \sec^2 t \cdot \frac{1}{at \cdot \cos t} \quad \left[\because \frac{dx}{dt} = at \cos t, \quad \therefore \frac{dt}{dx} = \frac{1}{at \cos t} \right]$$

$$= \frac{\sec^3 t}{at}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}$$

15. Find the value of K so that the following function f is continuous at the indicated point.

$$f(x) = \begin{cases} \frac{K \cos x}{x-2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$$

Solution :

$$\text{Left Hand Limit} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x)$$

$$= \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) \quad \left[\text{Putting } x = \frac{\pi}{2} - h \right]$$

$$= \lim_{h \rightarrow 0} \frac{K \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{K \sin h}{\pi - \pi + 2h}$$

$$= \lim_{h \rightarrow 0} \frac{K \sin h}{2h}$$

$$= \lim_{h \rightarrow 0} \frac{K}{2} \cdot \frac{\sin h}{h}$$

$$= \frac{K}{2} \cdot 1 \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{K}{2}$$

$$\text{Right Hand Limit} = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x)$$

$$= \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} + h\right) \quad \left[\text{Putting } x = \frac{\pi}{2} + h \right]$$

$$= \lim_{h \rightarrow 0} \frac{K \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-K \sin h}{\pi - 2h - \pi}$$

$$= \lim_{h \rightarrow 0} \frac{-K \sin h}{-2h}$$

$$= \lim_{h \rightarrow 0} \frac{K}{2} \cdot \frac{\sin h}{h}$$

$$= \frac{K}{2} \cdot 1$$

$$= \frac{K}{2}$$

$$\text{and } f\left(\frac{\pi}{2}\right) = 3$$

f is continuous at $x = \frac{\pi}{2}$ if RHL = LHL = functional value

$$\therefore \frac{K}{2} = 3$$

$$\text{or } K = 6$$

$\therefore f$ is continuous at $x = \frac{\pi}{2}$ if $K = 6$

16. Use differential to approximate $(26)^{\frac{1}{3}}$

Solution :

$$\text{Let } y = x^{\frac{1}{3}}$$

Let $x = 27$ and let $\Delta x = -1$. Then

$$\begin{aligned}\Delta y &= (x + \Delta x)^{\frac{1}{3}} - x^{\frac{1}{3}} \\ &= (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} \\ &= (26)^{\frac{1}{3}} - 3\end{aligned}$$

or, $(26)^{\frac{1}{3}} = 3 + \Delta y$

Now dy is approximately equal to Δy and is given by

$$\begin{aligned}dy &= \left(\frac{dy}{dx}\right)\Delta x \\ &= \frac{1}{3x^{\frac{2}{3}}}(-1) \quad \left(\text{as } y = x^{\frac{1}{3}}\right) \\ &= \frac{-1}{3\{(27)^{\frac{1}{3}}\}^2} \\ &= -\frac{1}{27} \\ &= -0.037\end{aligned}$$

Thus, approximate value of $(26)^{\frac{1}{3}}$ is given by $3 + (-0.037) = 2.963$

OR

Find the intervals in which function $f(x)$ is increasing or decreasing.

$$f(x) = 2x^3 - 15x^2 + 36x + 17$$

Solution :

We have,

$$f(x) = 2x^3 - 15x^2 + 36x + 17$$

$$\begin{aligned}\therefore f'(x) &= 6x^2 - 30x + 36 \\ &= 6(x^2 - 5x + 6) \\ &= 6(x-2)(x-3)\end{aligned}$$

$$\therefore f'(x) = 0 \text{ gives}$$

$$6(x-2)(x-3) = 0$$

or $(x-2)(x-3) = 0$

or $x = 2, 3$



The points 2 and 3 divide the real line into three disjoint intervals, namely $(-\infty, 2)$, $(2, 3)$, $(3, \infty)$.

In the intervals $(-\infty, 2)$ and $(3, \infty)$, $f'(x)$ is positive while in the interval $(2, 3)$, $f'(x)$ is negative.

$\therefore f$ is increasing in the interval $(-\infty, 2) \cup (3, \infty)$ and f is decreasing in the interval $(2, 3)$.

17. Evaluate $\int_0^{\pi} \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x}$

Solution :

$$\begin{aligned} \text{Let } I &= \int_0^{\pi} \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ &= \int_0^{\pi} \frac{(\pi - x)dx}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)} \quad \left[\because \int_0^a f(x) = \int_0^a f(a-x) \right] \\ &= \pi \int_0^{\pi} \frac{\pi dx}{a^2 \cos^2 x + b^2 \sin^2 x} - \int_0^{\pi} \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ &= \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} - I \end{aligned}$$

Thus $2I = \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$$\begin{aligned} \text{or } I &= \frac{\pi}{2} \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ &= \frac{\pi}{2} \cdot 2 \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \end{aligned}$$

$$\left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a-x) = f(x) \right]$$

$$= \pi \int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x} \quad (\text{dividing numerator and denominator by } \cos^2 x)$$

Put $b \tan x = t$, so that $b \sec^2 x dx = dt$.

Also, when $x = 0$, $t = 0$ and when $x = \frac{\pi}{2}$, $t \rightarrow \infty$ therefore,

$$I = \frac{\pi}{b} \int_0^{\infty} \frac{dt}{a^2 + t^2}$$

$$= \frac{\pi}{b} \cdot \frac{1}{a} \left[\tan^{-1} \frac{t}{a} \right]_0^{\infty}$$

$$= \frac{\pi}{ab} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi^2}{2ab}$$

$$\therefore \int_0^{\pi} \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$$

18. $I = \int x \tan^{-1} x dx$

Integrating by parts taking $\tan^{-1} x$ as 1st function), we get –

$$I = \tan^{-1} x \int x dx - \int \left\{ \frac{d}{dx} (\tan^{-1} x) \int x dx \right\} dx$$

$$= \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx + C_1$$

$$= \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1+x^2} dx + C_1$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{x^2 + 1} + C_1$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + C_1 + C_2$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C \text{ where } C = C_1 + C_2$$

OR

$$I = \int \frac{x}{\sqrt{8+x-x^2}} dx = \int \frac{-\frac{1}{2}(1-2x) + \frac{1}{2}}{\sqrt{8+x-x^2}}$$

$$= -\int \frac{x}{\sqrt{x^2-x-8}} dx = -\frac{1}{2} \int \frac{1-2x}{\sqrt{8+x-x^2}} dx + \frac{1}{2} \int \frac{dx}{\sqrt{8+x-x^2}}$$

$$= -\frac{1}{2} \int \frac{1-2x}{\sqrt{x^2-x-8}} dx + \frac{1}{2} \int \frac{dx}{\sqrt{8+x-x^2}}$$

Putting $8 + x - x^2 = z$ and therefore, $(1 - 2x)dx = dz$ in 1st integral

$$\begin{aligned}
 &= -\frac{1}{2} \int \frac{dz}{\sqrt{z}} + \frac{1}{2} \int \frac{dx}{\sqrt{-(x^2 - x - 8)}} \\
 &= -\frac{1}{2} \frac{dz}{\sqrt{z}} + \frac{1}{2} \int \frac{1}{\sqrt{\left[-x \frac{1}{2} x + \frac{1}{4} - \frac{1}{4} - 8\right]}} dx \\
 &= -\sqrt{z} + \frac{1}{2} \int \frac{1}{\sqrt{\left(\frac{\sqrt{33}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}} dx \\
 &= -\sqrt{8 + x - x^2} + \frac{1}{2} \sin^{-1} \frac{2x - 1}{\sqrt{33}} + C \\
 I &= -\sqrt{x^2 - x - x^2} + \frac{1}{2} \sin^{-1} \frac{2x - 1}{\sqrt{33}} + C
 \end{aligned}$$

19. $\int_{-1}^2 (7x - 5) dx$

We have

$$\int_a^b f(x) = \lim_{n \rightarrow \infty} h \left[f(a) + f(a+h) + \dots + f(a+(n-1)h) \right] \text{ where } h = \frac{b-a}{n}$$

Here, $b = 2$, $a = -1$, $f(x) = 7x - 5$.

$$\therefore f(a) = f(-1) = -12$$

$$f(a+h) = f(-1+h) = -12 + 7h$$

$$f(a+2h) = f(-1+2h) = -12 + 14h$$

$$f(a+(n-1)h) = f(-1+(n-1)h) = -12 + 7(nh - h)$$

$$\therefore I = \int_{-1}^2 (7x - 5) dx$$

$$= \lim_{h \rightarrow 0} h \left[-12 - 12 + 7h - 12 + 14h + \dots + (-12) + 14nh - 14h \right]$$

$$= \lim_{h \rightarrow 0} h \left[-12nh + \frac{7nh}{2}(nh - h) \right]$$

$$= \lim_{h \rightarrow 0} h \left[-12 \times 3 + \frac{7}{2}(3 - h) \right]$$

$$\begin{aligned}
&= -36 + \frac{63}{2} = \frac{-72 + 63}{2} \\
&= \frac{-9}{2}
\end{aligned}$$

OR

Solution :

We have,

$$\begin{aligned}
I &= \int \frac{1}{x^4 + 1} dx \\
\Rightarrow I &= \int \frac{1/x^2}{x^2 + \frac{1}{x^2}} dx \\
\Rightarrow I &= \frac{1}{2} \int \frac{\frac{2}{x^2}}{x^2 + \frac{1}{x^2}} dx \\
\Rightarrow I &= \frac{1}{2} \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} - \frac{1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx \\
\Rightarrow I &= \frac{1}{2} \int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx - \frac{1}{2} \int \frac{1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx \\
\Rightarrow I &= \frac{1}{2} \int \frac{1 + \frac{1}{x^2}}{\left(x^2 + \frac{1}{x}\right)^2 + 2} dx - \frac{1}{2} \int \frac{1 - \frac{1}{x^2}}{\left(x^2 + \frac{1}{x^2}\right)^2 - 2} dx
\end{aligned}$$

Putting $x - \frac{1}{x} = u$ in 1st integral and $x + \frac{1}{x} = v$ in 2nd integral, we get.

$$\begin{aligned}
I &= \frac{1}{2} \int \frac{du}{u^2 + (\sqrt{2})^2} - \frac{1}{2} \int \frac{dv}{v^2 - (\sqrt{2})^2} \\
&= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) - \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} \log \left| \frac{v - \sqrt{2}}{v + \sqrt{2}} \right| + C
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{2}} \right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x + \frac{1}{x} - \sqrt{2}}{x + \frac{1}{x} + \sqrt{2}} \right| + C \\
&= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right| + C
\end{aligned}$$

20. We have,

$$|\vec{a}| = 3$$

$$|\vec{b}| = 4$$

$$|\vec{c}| = 5$$

$$\text{and } \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\text{Now, } \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$|\vec{a} + \vec{b} + \vec{c}| = 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow 9 + 16 + 25 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow 50 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{50}{2}$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -25$$

21. We know that the shortest distance between the lines $\vec{\gamma} = \vec{a}_1 + \lambda \vec{b}_1$ & $\vec{\gamma} = \vec{a}_2 + \mu \vec{b}_2$ is given

$$\text{by } d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Comparing the above equation with

$$\vec{\gamma} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\& \vec{\gamma} = (s+1)\hat{i} + (25-1)\hat{j} - (25+1)\hat{k}$$

$$\therefore \vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \quad \therefore \vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\begin{aligned}\therefore \vec{a}_2 - \vec{a}_1 &= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k} \\ &= -\hat{j} - 4\hat{k}\end{aligned}$$

$$\begin{aligned}\therefore \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} \\ &= \hat{i}(-2+4) + \hat{j}(4) - 3\hat{k}\end{aligned}$$

$$\begin{aligned}\text{So, } (\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2) &= (-\hat{j} - 4\hat{k})(2\hat{i} + 4\hat{j} - 3\hat{k}) \\ &= -4 + 12 = 8\end{aligned}$$

$$\begin{aligned}|\vec{b}_1 \times \vec{b}_2| &= \sqrt{(2)^2 + (-4)^2 + (-3)^2} \\ &= \sqrt{29}\end{aligned}$$

$$\begin{aligned}\therefore \text{Shortest distance} &= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \\ &= \frac{8}{\sqrt{29}}\end{aligned}$$

23. Let E_1, E_2, E_3, E_4 & A be events

E_1 = missing card is heart

E_2 = missing card is spade

E_3 = missing card is club

E_4 = missing card is diamond

A = Drawing 2 diamond cards from the remaining cards.

$$P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = \frac{13}{52} = \frac{1}{4}, \quad P(E_3) = \frac{13}{52} = \frac{1}{4}, \quad P(E_4) = \frac{13}{52} = \frac{1}{4}$$

$P\left(\frac{A}{E_4}\right)$ = Probability of drawing a diamond cards given that one diamond lost

$$P\left(\frac{A}{E_4}\right) = \frac{{}^{12}C_2}{{}^{51}C_2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$P\left(\frac{A}{E_3}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

By Bayes theorem

$$\text{Required probability} = P\left(\frac{E_4}{A}\right)$$

$$= \frac{P(E_4)P\left(\frac{A}{E_4}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right) + P(E_4)P\left(\frac{A}{E_4}\right)}$$

$$= \frac{\frac{1}{4} \times \frac{{}^{12}C_2}{{}^{51}C_2}}{\frac{1}{4} \times \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{1}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2} + \frac{1}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2} + \frac{1}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2}}$$

$$= \frac{{}^{12}C_2}{{}^{12}C_2 + {}^{13}C_2 + {}^{13}C_2 + {}^{13}C_2}$$

$$= \frac{66}{66 + 78 + 78 + 78} = \frac{11}{50}$$

23. **Solution :** we write $A = IA$, i.e., $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

or $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ (applying $R_1 \leftrightarrow R_2$)

or $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$ (applying $R_3 \rightarrow R_3 - 3R_1$)

or
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \text{ (applying } R_1 \rightarrow R_1 - 2R_2)$$

or
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \text{ (applying } R_3 \leftrightarrow R_3 - 5R_2)$$

or
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_3 \rightarrow \frac{1}{2}R_3)$$

or
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_1 \rightarrow R_1 + R_3)$$

or
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_2 \rightarrow R_2 - 2R_3)$$

Here
$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

23. Let
$$A = \begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & -1 & 1+c \end{bmatrix}$$

Taking a, b & c common from C_1, C_2 & C_3

$$\Delta = abc \begin{bmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{bmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$= abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & 1+\frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$$= R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (abc) \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$= abc + bc + ac + ab$$

$$= abc + ab + bc + ca$$

$$24. \quad 3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

The above system of equation can be rewritten as –

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$A \quad X \quad = \quad B \text{ (Say)}$$

$$A^{-1} \quad A \quad X \quad = \quad A^{-1}B \rightarrow \text{(Operating } A^{-1} \text{ from left)}$$

$$X \quad = \quad A^{-1}B \quad \text{---(1)}$$

Co-factors can be denoted as A_{ij}

$$\therefore A_{11} = 2 - 3 = -1 \quad A_{12} = -(4 + 4) = -8 \quad A_{13} = (-6 - 4) = -10$$

$$A_{21} = -(-4 + 9) = -5 \quad A_{22} = 6 - 12 = -6 \quad A_{23} = -(-9 + 8) = 1$$

$$A_{31} = (2 - 3) = -1 \quad A_{32} = -(-3 - 6) = +9 \quad A_{33} = (3 + 4) = 7$$

$$|A| = 3(-1) - 8(-2) + 3(-10)$$

$$= -3 + 16 - 30$$

$$= 13 - 30 = -17$$

$$\text{Co-factors matrix } B = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$$

$$\text{Transpose of } B = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$\therefore X = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x = 1, y = 2, z = 3$$

25. A point of the hypotenuse of a right triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is

$$\left(a^{2/3} + b^{2/3}\right)^{3/2}$$

Solution :

Let AOB be a right, triangle with hypotenuse AB such that a point P on AB is at distances a and b from OA and OB respectively, i.e. $PL = a$ and $PM = b$

Let $\angle OAB = \theta$. Then,

$$AB = a \operatorname{cosec} \theta \quad BP = b \sec \theta$$

Let l be the length of the hypotenuse AB . Then,

$$l = AP + BP$$

$$\Rightarrow l = a \operatorname{cosec} \theta + b \sec \theta$$

$$\Rightarrow \frac{dl}{d\theta} = -a \operatorname{cosec} \theta \cot \theta + b \sec \theta \tan \theta$$

and,

$$\frac{d^2l}{d\theta^2} = a \operatorname{cosec}^3 \theta + a \operatorname{cosec} \theta \cot^2 \theta + b \sec^3 \theta + b \sec \theta \tan^2 \theta$$

For maximum or minimum, we must have

$$\frac{dl}{d\theta} = 0$$

$$\Rightarrow -a \operatorname{cosec} \theta \cot \theta + b \sec \theta \tan \theta = 0$$

$$\Rightarrow -\frac{a \cos \theta}{\sin^2 \theta} + \frac{b \sin \theta}{\cos^2 \theta} = 0$$

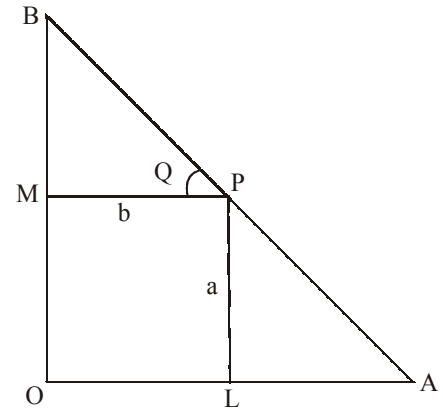
$$\Rightarrow \tan^3 \theta = \frac{a}{b}$$

$$\Rightarrow \tan \theta = \left(\frac{a}{b}\right)^{1/3}$$

$$\Rightarrow \sin \theta = \frac{a^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}} \quad \text{and} \quad \cos \theta = \frac{b^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}}$$

Clearly, $\frac{d^2l}{d\theta^2} > 0$ for $\tan \theta = \left(\frac{a}{b}\right)^{1/3}$

Thus, l is minimum when $\tan \theta = \left(\frac{a}{b}\right)^{1/3}$



The minimum value of l is given by

$$l = a \operatorname{cosec} \theta + b \sec \theta = a\sqrt{1 + \cot^2 \theta} + b\sqrt{1 + \tan^2 \theta}$$

$$\Rightarrow l = a\sqrt{1 + \left(\frac{b}{a}\right)^2} + b\sqrt{1 + \left(\frac{a}{b}\right)^2}$$

$$\Rightarrow l = \left(a^{2/3} + b^{2/3}\right)^{3/2}$$

26. Solution Let $R = \{(x, y) : y^2 \leq 4x^2 + 4y^2 \leq 9\}$
 $= \{(x, y) : y^2 \leq 4x\} \cap \{(x, y) : 4x^2 + 4y^2 \leq 9\}$
 $= R_1 \cap R_2$

where $R_1 = \{(x, y) : y^2 \leq 4x\}$ and $R_2 = \{(x, y) : 4x^2 + 4y^2 \leq 9\}$

Region R_1 : Clearly, $y^2 = 4x$ is the equation of the parabola with vertex at the origin along x -axis. Since we are given that $y^2 \leq 4x$, so R_1 is the region inside the parabola $y^2 = 4x$.

Region R_2 : We have, $4x^2 + 4y^2 = 9 \Rightarrow x^2 + y^2 = \left(\frac{3}{2}\right)^2$

Clearly, it represents a circle with centre at the origin and radius $\frac{3}{2}$.

It is given that $x^2 + y^2 \leq \frac{9}{4}$, so R_2 is the region lying inside the

circle $x^2 + y^2 = \left(\frac{3}{2}\right)^2$

Thus, the region R is the region bounded by

the parabola $y^2 = 4x$ and the circle $x^2 + y^2 = \left(\frac{3}{2}\right)^2$, as shown

by the shaded portion in figure.

To find the points of intersection of the given curves, we solve their

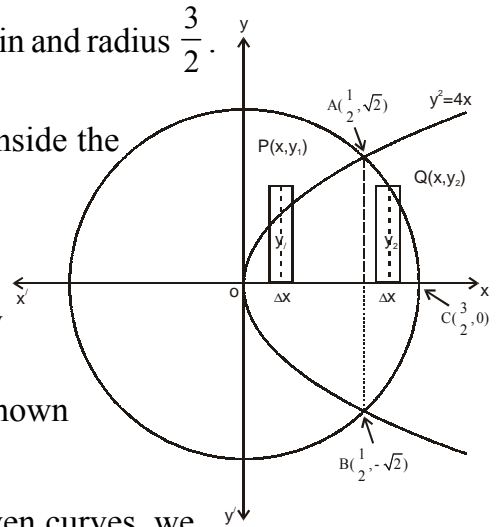
equations simultaneously.

We have, $y^2 = 4x$ ----- (i)

and, $4x^2 + 4y^2 = 9$ ----- (ii)

Putting $y^2 = 4x$ from (i) into (ii) we get

$$4x^2 + 16x = 9 \Rightarrow 4x^2 + 16x - 9 = 0 \Rightarrow (2x + 9)(2x - 1) = 0 \Rightarrow x = \frac{1}{2} \text{ or } x = -\frac{9}{2}$$



From (i), we find that

$$x = \frac{1}{2} \Rightarrow y = \pm\sqrt{2} \text{ and } x = -\frac{9}{2} \Rightarrow y \text{ is imaginary.}$$

So, the two curves intersect at $(\frac{1}{2}, \sqrt{2})$ and $(\frac{1}{2}, -\sqrt{2})$

Clearly, both the curves are symmetrical about x -axis.

So, Required area = 2(Area of the shaded region lying above x -axis).

To find this area, we slice it into vertical strips. We observe that vertical strips character their change at the point A we draw a line AD parallel to y -axis which divides the shaped region lying above x -axis into two portions $OADO$ and $ADCA$.

For the area $OADO$, the approximating rectangle has, length = y_1 , Width = Δx and Area = $y_1 \Delta x$.

As it can move from $x = 0$ to $x = \frac{1}{2}$

$$\begin{aligned} \text{Area } OADO &= \int_0^{\frac{1}{2}} y_1 dx \\ &= \int_0^{\frac{1}{2}} 2\sqrt{x} dx \end{aligned} \quad \left[\begin{array}{l} \because P(x, y_1) \text{ lies on } y^2 = 4x \\ \therefore y_1^2 = 4x \Rightarrow y_1 = 2\sqrt{x} \end{array} \right]$$

For the area $ADCA$, the approximating rectangle shown in figure.

Length = y_2 , Width = Δx and Area = $y_2 \Delta x$. As it can move from $x = \frac{1}{2}$ to $x = \frac{3}{2}$. So,

$$\begin{aligned} \text{Area } ADCA &= \int_{\frac{1}{2}}^{\frac{3}{2}} y_2 dx \\ &= \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4} - x^2} dx \end{aligned} \quad \left[\begin{array}{l} \because Q(x, y_2) \text{ lies on } 4x^2 + 4y^2 = 9 \\ \therefore 4x^2 + 4y_2^2 = 9 \Rightarrow y_2 = \sqrt{\frac{9}{4} - x^2} \end{array} \right]$$

Hence, Required area

$$\begin{aligned} &= 2[\text{Area } OADO + \text{Area } ADCA] \\ &= 2 \int_0^{\frac{1}{2}} 2\sqrt{x} dx + 2 \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4} - x^2} dx \\ &= 4 \cdot \frac{2}{3} [x^{3/2}]_0^{\frac{1}{2}} + 2 \left[\frac{1}{2} x \sqrt{\frac{9}{4} - x^2} + \frac{1}{2} \cdot \frac{9}{4} \sin^{-1} \frac{2x}{3} \right]_{\frac{1}{2}}^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned}
&= \frac{8}{3} \left(\frac{1}{2\sqrt{2}} - 0 \right) + \left[\left\{ \frac{9}{4} \sin^{-1}(1) \right\} - \left\{ \frac{1}{\sqrt{2}} + \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) \right\} \right] \\
&= \frac{2\sqrt{2}}{3} + \left[\frac{9}{8} \pi - \frac{1}{\sqrt{2}} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) \right] \\
&= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) \right] \text{sq. units}
\end{aligned}$$

27. Let L be the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line. The coordinates of general point on

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{2} \text{ are given by}$$

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{2} = \lambda$$

i.e. $x = 5\lambda - 3, y = 2\lambda + 1, z = 2\lambda - 4$

Let the coordinates of L be

$$(5\lambda - 3, 2\lambda + 1, 2\lambda - 4) \text{ ----- (1)}$$

\therefore Direction ratios of PL are proportional to

$$5\lambda - 3 - 0, 2\lambda + 1 - 2, 2\lambda - 4 - 3$$

i.e. $5\lambda - 3, 2\lambda - 1, 2\lambda - 7$

Direction ratios of given line are proportional to 5, 2 and 2

\therefore PL is perpendicular to given line

$$\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 2(2\lambda - 7) = 0$$

28. **Solution :** Let the cereal contain x kg of bran and y kg of rice. According to the hypotheses, the L.P.P. is Minimize $Z = 5x + 4y$. Subject to the constraints

$$x \times \frac{80}{1000} + y \times \frac{100}{1000} \geq \frac{88}{1000}$$

$$\Rightarrow 20x + 25y \geq 22$$

$$x \times \frac{40}{1000} + y \times \frac{30}{1000} \geq \frac{36}{1000}$$

$$\Rightarrow 20x + 15y \geq 18$$

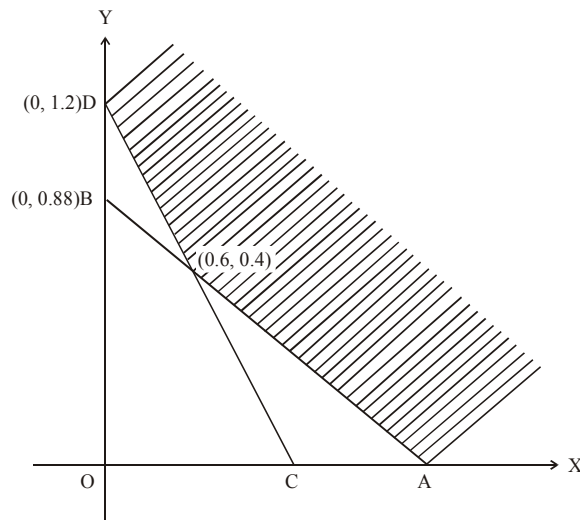
$$x \geq 0, y \geq 0$$

Now draw the lines

$$AB : 20x + 25y = 22$$

and $CD : 20x + 15y = 18$

These lines meet at $E(0.6, 0.4)$. The feasible region has been shaded and it is an unbounded region with vertices, A , E and D .



At $A(1, 1, 0)$, the cost is

$$Z = 5 \times 1.1 + 4 \times 0.4 = 5.5$$

At $E(0.6, 0.2)$, the cost is

$$Z = 5 \times 0.6 + 4 \times 0.4 = 4.6$$

At $D(0, 1.4)$, the cost is

$$Z = 5 \times 0 + 4 \times 1.2 = 4.8$$

\therefore the minimum cost of producing this cereal is Rs. 4.6.

29. Let X denote the number of kings in a draw of two cards. X is a random variable which can assume the values 0, 1 or 2.

$$\text{Now, } P(X = 0) = P(\text{no king}) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{\frac{48!}{2!(48-2)!}}{\frac{52!}{2!(52-2)!}}$$

$$= \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$$P(X = 1) = P(\text{one king \& one non-king}) = \frac{{}^4C_1 {}^{48}C_1}{{}^{52}C_2}$$

$$= \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}$$

$$\& P(X = 2) = P(\text{two kings}) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

Thus, probability distribution of X is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$\text{Mean of } X, E(x) = \sum_{i=1}^n x_i P(x_i)$$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221} = \frac{34}{221}$$

$$\text{Also, } X, E(x^2) = \sum_{i=1}^n x_i^2 P(x_i) = 1 \times \frac{32}{221} + 4 \times \frac{1}{221} = \frac{36}{221}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{(221)^2}$$

$$\therefore \sigma_x = \sqrt{\text{Var}(X)} = \frac{\sqrt{6800}}{221} = 0.37$$

MODEL TEST PAPER-2

Time Allowed : 3 hrs.

Max. Marks : 100

Part-A

- Let $f : R \rightarrow R$ be defined as $f(x) = x^4$. Choose the correct answer.
(A) f is one-one onto (B) f is many one onto
(C) f is one-one but not onto (D) f is neither one-one nor onto
- $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to –
(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$
- Let A be a non-singular matrix of order 3×3 . Then $|adjA|$ is equal to –
(A) $|A|$ (B) $|A|^2$
(C) $|A|^3$ (D) $3|A|$
- The interval in which $y = x^2e^{-x}$ is increasing –
(A) $(-\infty, \infty)$ (B) $(-2, 0)$ (C) $(2, \infty)$ (D) $(0, 2)$
- Differentiate $\sin(\tan^{-1}e^x)$ w.r.t. $-x$.
- The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is –
(A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x
- Evaluate $\int \sqrt{1 + \cos 2x} dx$.
- Let the vectors \vec{a} and \vec{b} be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between \vec{a} and \vec{b} is –
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
- Find the vector equation of the line passing through $(-1, 0, 2)$ and $(3, 4, 6)$.
- Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

Part-B

11. Determine whether the following relation is reflexive, symmetric and transitive.

$$R = \{(x, y) : y \text{ is divisible by } x\}$$

$$x, y \in A \quad \text{where } A = \{1, 2, 3, 4, 5, 6\}$$

12. Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

13. Solve the differential equation

$$(x + y)dy + (x - y)dx = 0$$

Solve the differential equation

$$ydx - (x + 2y^2)dy = 0$$

14. Find all points of discontinuity of f where

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x & \text{if } x \geq 0 \\ x + 1 & \end{cases}$$

15. If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$ show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

16. Show that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$

OR

Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to the line $y - 4x + 5 = 0$.

17. Evaluate $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\cot x}}$

18. Evaluate $\int \frac{1 dx}{\sqrt{(x-a)(x-b)}}$

$$\int \frac{3x + 2}{(x-1)(2x+3)} dx$$

19. Evaluate $\int \frac{x^2 + 1}{x^4 + x^2 + 1} dx$

$$\int \sec^3 x dx$$

20. If $\hat{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\hat{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\hat{a} + \hat{b}$ and $\hat{a} - \hat{b}$ are orthogonal
21. Find the image of point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
22. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Part-C

23. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

OR

using properties of determinants show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2ab \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

24. Solve the following system of the equations using matrix method.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

25. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.
26. Find the area of the region enclosed between two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.
27. Find the cartesian as well as the vector equation of the planes passing through the intersection of the planes

$$\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 1$$

and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0,$

which are at unit distance from the origin.

28. A manufacturer of medicines is preparing a production plan of medicines M_1 and M_2 . There are sufficient raw materials available to make 20000 bottles of M_1 and 40000 bottles of M_2 , but there are only 45000 bottles into which either of the medicines can be put. Further, it takes 3 hours to prepare enough material to fill 1000 bottles of M_1 , it takes 1 hour to prepare enough material to fill 1000 bottles of M_2 and there are 66 hours available for this operation. The profit is Rs 8 per bottle for M_1 and Rs. 7 per bottle for M_2 . How should the manufacturer schedule his/her production in order to maximise profit?
29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

SOLUTION SET-II

1. (D)
2. (B)
3. (B)
4. (D)
5. $\frac{e^x \cos(\tan^{-1} e^x)}{1+e^{2x}}$
6. (C)
7. $\sqrt{2} \sin x + C$
8. (B)
9. Let \vec{a} and \vec{b} be position vectors of $(-1, 0, 2)$ and $(3, 4, 6)$

$$\therefore \vec{a} = -\vec{c} + 2\hat{k}, \quad \vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\text{and} \quad \vec{b} - \vec{a} = 4\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\text{Required line is } \vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

$$10. \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{1}{\sqrt{6}} \{ \hat{i} + \hat{j} + 2\hat{k} \}$$

2. **Soltion :**

$$\cos^{-1} \left(\cos \frac{7\pi}{6} \right)$$

We know the range of \cos^{-1} function is $[0, \pi]$

$$\text{So,} \quad \cos^{-1} \left(\cos \frac{7\pi}{6} \right) = \cos^{-1} \left\{ \cos \left(2\pi - \frac{5\pi}{6} \right) \right\}$$

$$= \cos^{-1} \left\{ \cos \frac{5\pi}{6} \right\}$$

$$= \frac{5\pi}{6}$$

4. **Solution :**

$$f(x) = x^2 e^{-x}$$

$$\therefore f'(x) = 2xe^{-x} + x^2(-e^{-x})$$

$$= xe^{-x}(2-x) = e^{-x} \cdot x(2-x)$$

Now, e^{-x} is positive for all $x \in R$.

$$f'(x) = 0 \text{ at } x = 0, 2$$

$x = 0$ and $x = 2$ divide the number line into three disjoint intervals, $(-\alpha, 0)$, $(0, 2)$, $(2, \alpha)$

(a) Interval $(-\alpha, 0)$.

x is -ve and $(2-x)$ is +ve

$$f'(x) = e^{-x} \cdot x(2-x) = (+)(-)(+) = -ve$$

f is decreasing in $(-\alpha, 0)$.

(b) Interval $(0, 2)$.

$$f'(x) = e^{-x} \cdot x(2-x)$$

$$= (+)(+)(+) = +ve$$

f is increasing in $(0, 2)$

(c) Interval $(2, \alpha)$

$$f'(x) = e^{-x} \cdot x(2-x)$$

$$= (+)(+)(-) = -ve$$

So, f is decreasing in the interval $(2, \alpha)$.

5. **Solution :**

$$\text{Let } y = \sin(\tan^{-1} e^x)$$

Differentiating both sides w.r.t. x ,

$$\therefore \frac{dy}{dx} = \cos(\tan^{-1} e^x) \cdot \frac{d}{dx}(\tan^{-1} e^x)$$

$$= \cos(\tan^{-1} e^x) \frac{1}{1+(e^x)^2} \frac{d}{dx}(e^x)$$

$$\begin{aligned}
&= \cos(\tan^{-1} e^x) \frac{1 \times e^x}{1 + e^{2x}} \\
&= \frac{e^x \cos(\tan^{-1} e^x)}{1 + e^{2x}}
\end{aligned}$$

11. **Solution :** Given, $R = \{(x, y) : y \text{ is divisible by } x\}$

$$A = \{1, 2, 3, 4, 5, 6\} \text{ and } x, y \in A$$

$$\therefore R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (2, 4), (2, 6), (3, 6)\}$$

12. **Solution :** $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$

$$= \tan^{-1} \left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2}} \right]$$

$$= \tan^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2} \right]$$

$$= \tan^{-1} \left[\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)} \right]$$

$$= \tan^{-1} \left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right]$$

$$= \tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] \quad \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$= \frac{\pi}{4} + \frac{x}{2}$$

13. The given differential equation can be written as

$$\frac{dy}{dx} = -\frac{x-y}{x+y}$$

or
$$\frac{dy}{dx} = +\frac{y-x}{x+y}$$

Since each of the functions $y - x$ and $y + x$ is a homogeneous function of degree 1. Therefore, equation (i) is a homogeneous differential equation.

Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ in (i), we get

$$\begin{aligned} v + x \frac{dv}{dx} &= \frac{vx-x}{x+vx} \\ \Rightarrow v + x \frac{dv}{dx} &= \frac{v-1}{v+1} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v-1}{v+1} - v \\ \Rightarrow x \frac{dv}{dx} &= -\left(\frac{v^2+1}{v+1}\right) \\ \Rightarrow \frac{v+1}{v^2+1} dv &= -\frac{dx}{x} \quad \text{[By separating the variables]} \\ \Rightarrow \int \frac{v+1}{v^2+1} dv &= -\int \frac{dx}{x} \quad \text{[Integrating both sides]} \\ \Rightarrow \int \frac{v}{v^2+1} dv + \int \frac{1}{v^2+1} dv &= -\int \frac{dx}{x} \\ \Rightarrow \frac{1}{2} \int \frac{2v}{v^2+1} dv + \int \frac{1}{v^2+1} dv &= -\int \frac{dx}{x} \\ \Rightarrow \frac{1}{2} \log(v^2+1) + \tan^{-1} v &= -\log|x| + C. \\ \Rightarrow \log(v^2+1) + 2 \log|x| + 2 \tan^{-1} v &= 2C. \\ \Rightarrow \log\left|(v^2+1)x^2\right| + 2 \tan^{-1} v &= k \\ \Rightarrow \log\left|(y^2/x^2+1)x^2\right| + 2 \tan^{-1}(y/x) &= k \quad [\because v = y/x] \\ \Rightarrow \log(x^2+y^2) + 2 \tan^{-1}(y/x) &= k \dots (ii) \end{aligned}$$

It is given that $y = 1$, where $x = 1$. Putting $x = 1$, $y = 1$ in (ii), we get

$$\log 2 + 2 \tan^{-1}(1) = k \Rightarrow \log 2 + 2(\pi/4) = (\pi/2) + \log 2$$

Substituting the value of k in (ii), we get

$$\log(x^2 + y^2) + 2 \tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$$

Hence, $\log(x^2 + y^2) + 2 \tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2, x \neq 0$

is the required solution of the given differential equation.

OR

Solve : $ydx - (x + 2y^2)dy = 0$

Solution : The given differential equation is –

$$ydx - (x + 2y^2)dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x + 2y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + 2y^2}{y}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{-1}{y}\right) = 2y$$

This is a linear differential equation of the form

$$\frac{dx}{dy} + Rx = S, \text{ where } R = -\frac{1}{y} \text{ and } S = 2y$$

$$\therefore \text{I.F.} = e^{\int R dy} = e^{\int \frac{-1}{y} dy} = e^{-\log y} = e^{\log y^{-1}} = y^{-1}$$

Multiplying both sides of (i) by I.F. y^{-1} , we obtain

$$\frac{1}{y} \frac{dx}{dy} - \frac{1}{y^2} x = 2$$

$$\therefore \frac{x}{y} = 2y + C \text{ is the required solution}$$

14. Solution :

$$f(x) = \begin{cases} \frac{\sin x}{x} & x < 0 \\ x+1 & x \geq 0 \end{cases}$$

Continuity at $x = 0$,

Left hand limit

$$\begin{aligned}\lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left(\frac{\sin x}{x} \right) \\ \therefore \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left(\frac{\sin x}{x} \right) \\ \therefore \lim_{h \rightarrow 0} f(0-h) &= \lim_{h \rightarrow 0} \left(\frac{\sin(0-h)}{(0-h)} \right) \\ &= \lim_{h \rightarrow 0} \frac{\sin(-h)}{(-h)} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h}\end{aligned}$$

Right hand limit at $x = 0$.

$$\begin{aligned}\lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (x+1) \\ \therefore \lim_{h \rightarrow 0} f(0+h) &= \lim_{h \rightarrow 0} (x+1) \\ &= \lim_{h \rightarrow 0} (h+1) \\ &= 0+1 \\ &= 1\end{aligned}$$

\therefore left hand limit = right hand limit

So, f is a continuous function

There is no points of discontinuity.

15. **Solution :** $y = e^{a \cos^{-1} x}$

\therefore Differentiating both sides *w.r.t.* x ,

$$\begin{aligned}\therefore \frac{dy}{dx} &= \frac{d}{dx} \left(e^{a \cos^{-1} x} \right) \\ &= e^{a \cos^{-1} x} \cdot \frac{d}{dx} (a \cos^{-1} x) \\ &= a e^{a \cos^{-1} x} \cdot a \frac{d}{dx} (\cos^{-1} x) \\ &= \frac{-a e^{a \cos^{-1} x}}{\sqrt{1-x^2}} = \frac{-ay}{\sqrt{1-x^2}} \quad [\text{given}] \text{-----(i)}\end{aligned}$$

Again differentiating both both sides *w.r.t.* x ,

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left[\frac{-ay}{\sqrt{1-x^2}} \right]$$

$$\begin{aligned}
&= -a \frac{d}{dx} \left(\frac{y}{\sqrt{1-x^2}} \right) \\
&= -a \left\{ \frac{\sqrt{1-x^2} \frac{dy}{dx} - y \frac{d}{dx} (\sqrt{1-x^2})}{(\sqrt{1-x^2})^2} \right\} \\
&= -a \left\{ \frac{\sqrt{1-x^2} \cdot \left(\frac{-ay}{\sqrt{1-x^2}} \right) - y \cdot \frac{1(-2x)}{2\sqrt{1-x^2}}}{1-x^2} \right\} \\
&= -a \left\{ \frac{-ay + xy/\sqrt{1-x^2}}{1-x^2} \right\} \\
&= -a \left\{ \frac{-ay\sqrt{1-x^2} + xy}{\sqrt{1-x^2}(1-x^2)} \right\} \\
\therefore (1-x^2) \cdot \frac{d^2 y}{dx^2} &= \frac{a^2 y \sqrt{1-x^2} - axy}{\sqrt{1-x^2}} \\
&= a^2 y - \frac{axy}{\sqrt{1-x^2}} \\
&= a^2 y + \frac{xdy}{dx} \quad \text{[From (i)]} \\
\therefore (1-x^2) \cdot \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y &= 0, \text{ Hence proved.}
\end{aligned}$$

16. Solution :

The given curves are –

$$x = y^2 \text{ ----- (1)}$$

$$xy = K \text{ ----- (2)}$$

Puttiing $x = y^2$ is (2) we get

$$y^3 = K \Rightarrow y = K^{2/3}$$

Thus, the point of intersection is $(K^{2/3}, K^{1/3})$. Differentiating (1) *w.r.t.* x , we have

$$1 = 2y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$\text{At } \left(K^{2/3}, K^{1/3}\right), \frac{dy}{dx} = \frac{1}{2K^{1/3}}$$

\therefore Slope of the tangent at $\left(K^{2/3}, K^{1/3}\right)$

$$= \frac{1}{2K^{1/3}}$$

Differentiating $xy = K$ w.r.t. x , we get

$$x \frac{dy}{dx} + y = 0 \Rightarrow x \frac{dy}{dx} = -y$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x}$$

$$\text{At } \left(K^{2/3}, K^{1/3}\right), \frac{dy}{dx} = \frac{-K^{1/3}}{K^{2/3}} = \frac{-1}{K^{1/3}}$$

$$\Rightarrow \text{Slope of the tangent at } \left(K^{2/3}, K^{1/3}\right) = \frac{-1}{K^{1/3}}$$

Now, the curves cut at right angles if the product of slopes of tangents to two curves is -1 .

$$i.e. \text{ if } \left(\frac{1}{2K^{1/3}}\right) \left(\frac{-1}{K^{1/3}}\right) = -1$$

$$\Rightarrow +1 = +2K^{2/3}$$

$$\therefore (-1)^3 = 8K^2 \Rightarrow 8K^2 = 1, \text{ Hence proved.}$$

17. Solution :

$$\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\cot x}}$$

$$\text{or } \int_0^{\pi/2} \frac{dx}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$$

$$= \int_0^{\pi/2} \frac{dx}{\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x}}}$$

$$\therefore I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \quad \text{-----(1)}$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x \left(\frac{\pi}{2} - x \right)}}{\sqrt{\sin \left(\frac{\pi}{2} - x \right) + \sqrt{\cos \left(\frac{\pi}{2} - x \right)}} dx \quad \left[\int_0^a f(x) = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \quad \text{-----(2)}$$

Adding (1) and (2)

$$\begin{aligned} 2I &= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sqrt{\sin x}}} \\ &= \int_0^{\pi/2} \frac{\sqrt{\sin x + \sqrt{\cos x}}}{\sqrt{\cos x + \sqrt{\sin x}}} \\ &= \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2} \end{aligned}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

18. Solution :

$$\begin{aligned} I &= \int \frac{dx}{\sqrt{(x-a)(x-b)}} \\ &= \int \frac{dx}{\sqrt{x^2 - (a+b)x + ab}} \\ &= \int \frac{dx}{\sqrt{\left[x^2 - 2\left(\frac{a+b}{2}\right)x + \left(\frac{ab}{2}\right)^2 \right] + ab - \left(\frac{a+b}{2}\right)^2}} \\ &= \int \frac{dx}{\sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2}} \end{aligned}$$

$$\begin{aligned}
&= \log \left| \left(x - \frac{a+b}{2} \right) + \sqrt{\left(x - \frac{a+b}{2} \right)^2 - \left(\frac{a-b}{2} \right)^2} \right| + C \\
&= \log \left| x - \frac{a+b}{2} + \sqrt{(x-a)(x-b)} \right| + C
\end{aligned}$$

OR

$$\int \frac{3x+2dx}{(x-1)(2x+3)}$$

Let $\frac{3x+2}{(x-1)(2x+3)} = \frac{A}{x-1} + \frac{B}{2x+3}$

$$\begin{aligned}
\therefore 3x+2 &= A(2x+3) + B(x-1) \\
&= 2Ax + 3A + Bx - B \\
&= (2A+B)x + (3A-B)
\end{aligned}$$

\therefore Comparing the co-efficients

$$2A + B = 3$$

$$\therefore B = 3 - 2A \text{ ----- (1)}$$

and $3A - B = 2$

$$\Rightarrow 3A - (3 - 2A) = 2$$

$$\Rightarrow 5A = 5$$

$$\Rightarrow A = 1$$

$$\therefore B = 3 - 2 = 1$$

$$\therefore \int \frac{(3x+1)dx}{(x-1)(2x+3)}$$

$$= \int \left(\frac{1}{x-1} + \frac{1}{2x+3} \right) dx$$

$$= \int \frac{1}{(x-1)} dx + \int \frac{1dx}{(2x+3)}$$

$$= \log|x-1| + C_1 + \frac{1}{2} \int \frac{2dx}{2x+3}$$

$$= \log|x-1| + C_1 + \frac{1}{2} \log|2x+3| + C_2$$

$$= \log|x-1| + \frac{1}{2} \log|2x+3| + C$$

[where $C = C_1 + C_2$]

19. Solution :

$$\int \frac{x^2+1}{x^4+x^2+1} \cdot dx$$

Let $I = \int \frac{x^2+1}{x^4+x^2+1} \cdot dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{1+\frac{1}{x^2}}{1+x^2+\frac{1}{x^2}} \cdot dx$$

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+(\sqrt{3})^2} \cdot dx$$

Let $\frac{1}{x} - x = u$ or $\frac{1}{x} - x = u$

Then $\frac{d\mu}{dx} = \left(1 + \frac{1}{x^2}\right)$ [Differentiating both sides w.r.t x]

$$\therefore du = \left(1 + \frac{1}{x^2}\right) dx$$

$$\therefore I = \int \frac{1+\frac{1}{x^2} dx}{\left(x-\frac{1}{x}\right)^2+(\sqrt{3})^2}$$

$$= \int \frac{du}{u^2+(\sqrt{3})^2}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) + C$$

OR

Solution :

$$\begin{aligned} & \int \sec^2 x dx \\ &= \int \sec x \sec^2 x dx \\ &= \sec x \int \sec^2 x dx - \int \left\{ \frac{d}{dx} (\sec x) \right\} \left\{ \int \sec^2 x dx \right\} dx \\ &= \sec x \tan x - \int \sec x \tan x \tan x dx \\ &= \sec x \tan x - \int \sec x \tan^2 x dx \\ &= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx \\ &= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx \\ \therefore 2 \int \sec^3 x dx &= \sec x \tan x + \log |\sec x + \tan x| + C \\ \therefore \int \sec^3 x dx &= \frac{1}{2} \sec x \tan x + \frac{1}{2} \log |\sec x + \tan x| + C \end{aligned}$$

20. Solution :

$$\text{Given, } \vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$$

$$\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$$

$$\text{Now, } \vec{a} + \vec{b} = (5\hat{i} - \hat{j} - 3\hat{k}) + (\hat{i} + 3\hat{j} - 5\hat{k})$$

$$= 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{a} - \vec{b} = (5\hat{i} - \hat{j} - 3\hat{k}) - (\hat{i} + 3\hat{j} - 5\hat{k})$$

$$= 4\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\text{So, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = (6\hat{i} + 2\hat{j} - 8\hat{k}) \cdot (4\hat{i} - 4\hat{j} + 2\hat{k})$$

$$= 24 - 8 - 16 = 0$$

Hence, $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal

21. Solution : Given lines are

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} \quad \text{----- (1)}$$

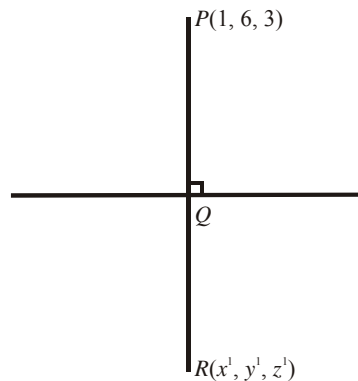
For image of $P(1, 6, 3)$ in line (1) draw a line PR^{\wedge} to line (1), the R is its image if Q is mid point of PR and PR^{\wedge} to line (1).

Let λ, μ, ν be the d.r.'s. of PR and PR^\perp to line (1) then

$$\begin{aligned} \lambda \times 1 + \mu \times 2 + \nu \times 3 &= 0 \\ \Rightarrow \lambda + 2\mu + 3\nu &= 0 \end{aligned} \quad \text{----- (2)}$$

Also equation of PR is

$$\frac{x-1}{\lambda} = \frac{y-6}{\mu} = \frac{z-3}{\nu} (=k) \quad \text{----- (3)}$$



Any point on line (2) is

$$(\lambda k + 1, \mu k + 6, \nu k + 3)$$

Let it be Q .

As Q lies on line (1), so

$$\begin{aligned} \frac{\lambda k + 1}{1} &= \frac{\mu k + 6 - 1}{2} = \frac{\nu k + 3 - 2}{3} \\ \Rightarrow \frac{\lambda k + 1}{1} &= \frac{\mu k + 5}{2} = \frac{\nu k + 1}{3} \\ &= \frac{1(\lambda k + 1) + 2(\mu k + 5) + 3(\nu k + 1)}{1 \times 1 + 2 \times 2 + 3 \times 3} \\ &= \frac{14 + (\lambda + 2\mu + 3\nu)k}{14} = 1; \end{aligned} \quad \text{[using (2)]}$$

$$\Rightarrow \lambda k = 0, \mu k = -3, \nu k = 2$$

$$\Rightarrow Q = (0 + 1, -3 + 6.2 + 3) \text{ or } (1, 3, 5)$$

As Q is the mid point of PR , so

$$\frac{1+x'}{2} = 1, \frac{6+y'}{2} = 3, \frac{3+z'}{2} = 5$$

$$\Rightarrow x' = 1, y' = 0, z' = 7$$

$$\Rightarrow R(1, 0, 7), \text{ which is the image of } P \text{ in line (1)}$$

22. **Solution :** Let E be the event that the man reports that six occurs in the throwing of the die and let S_1 be the event that six occurs and S_2 be the event that six does not occur.

$$\text{Then } P(S_1) = \text{probability that six occurs} = \frac{1}{6}$$

$$P(S_2) = \text{probability that six does not occur} = \frac{5}{6}$$

$P(E|S_1)$ = probability that the man report that six occurs when six had actually occurred on the die.

$$= \text{probability that the man speaks the truth} = \frac{3}{4}$$

$P(E|S_2)$ = Probability that the man reports that six occurs when six has not actually occurred on the die.

$$= \text{Probability that the man does not speaks the truth} = 1 - \frac{3}{4} = \frac{1}{4}$$

Thus by Bage's theorem, we get

$P(S_2|E)$ = Probability that the report of the man that six has occurred is actually a six.

$$= \frac{P(S_1) P(E|S_1)}{P(S_1) P(E|S_1) + P(S_2) P(E|S_2)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$$

$$= \frac{1}{8} \times \frac{24}{8} = \frac{3}{8}$$

Hence, the required probability = $\frac{3}{8}$

23. **Solution :**

Given,

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

We know, $A = AI$

$$i.e. \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\text{or } \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (P_1 \leftrightarrow P_2)$$

$$\text{or } \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad (P_3 \leftrightarrow P_3 - 3P_1)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad (P_1 \leftrightarrow P_1 - 2P_1)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \quad (R_3 \leftrightarrow R_3 + 5R_2)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad \left(R_3 \leftrightarrow \frac{1}{2} R_3 \right)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 3 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad (R_1 \leftrightarrow R_1 + R_3)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad (R_2 \leftrightarrow R_2 + 2R_3)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

OR

Solution :

$$\text{L.H.S.} = A = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 + bR_3$

$$\begin{aligned} D &= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\ &= (1+a^2+b^2) \begin{vmatrix} 1 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\ &= (1+a^2+b^2) \begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \end{aligned}$$

Operating $R_1 \rightarrow R_2 - aR_3$

$$\begin{aligned} &= (1+a^2+b^2) \begin{vmatrix} 1 & 0 & -b \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\ &= (1+a^2+b^2) \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \end{aligned}$$

On expanding we get

$$\begin{aligned}
 D &= (1 + a^2 + b^2)^2 \{1(1 - a^2 - b^2 + 2a^2) - b(0 - 2b)\} \\
 &= (1 + a^2 + b^2)^2 \{(1 + a^2 + b^2)\} \\
 &= (1 + a^2 + b^2)^2 (1 + a^2 + b^2) \\
 &= (1 + a^2 + b^2)^3 \\
 &= RHS, \text{ Hence proved}
 \end{aligned}$$

24. Solution :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Let $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

\therefore System of equation is

$$2u + 3v + 10w = 4$$

$$4u - 6v + 5w = 1$$

$$6u + 9v - 20w = 2$$

Which may be written as

$$AX = B$$

Where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

To find $adj A$, we find the cofactors of the elements

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 5 \\ 9 & -20 \end{vmatrix} = 120 - 45 = 75$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = -(-80 - 30) = 110$$

$$A_{13} = (-1)^{1+3} = \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} = 36 + 36 = 72$$

$$A_{21} = (-1)^{2+1} = \begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = -(-60 - 90) = 150$$

$$A_{22} = (-1)^{2+2} = \begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} = -40 - 60 = -100$$

$$A_{23} = (-1)^{2+3} = \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = -(18 - 18) = 0$$

$$A_{31} = (-1)^{3+1} = \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = 15 + 60 = 75$$

$$A_{32} = (-1)^{3+2} = \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = -(10 - 40) = 30$$

$$A_{33} = (-1)^{3+3} = \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = -12 - 12 = -24$$

$$\therefore \text{Adj}A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

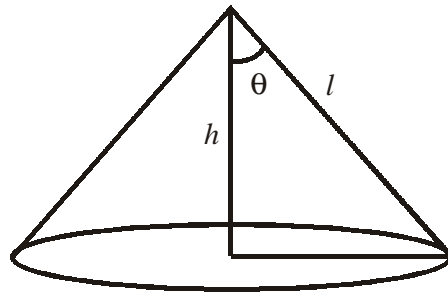
$$|A| = 2 \times 75 + 3 \times 110 + 10 \times 72 \\ = 150 + 330 + 720 = 1200$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\begin{aligned} \therefore X = A^{-1} \cdot B &= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \\ &= \frac{1}{1200} \begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix} \quad \therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5} \\ &\quad \therefore x = 2, y = 3, z = 5 \end{aligned}$$

25. **Solution :**



Let v be the volume, l be the slant height and θ be the semi-vertical angle of the cone.

$$\text{Volume of the cone} = V = \frac{1}{3} \pi r^2 h$$

$$\text{Vertical height (h)} = l \sin \theta$$

$$\text{and radius (r)} = l \cos \theta$$

$$\therefore V = \frac{1}{3} \pi (l \cos \theta)^2 (l \sin \theta)$$

$$= \frac{1}{3} \pi l^3 \cos^2 \theta \sin \theta$$

$$\therefore \frac{dV}{d\theta} = \frac{1}{3} \pi l^3 (2 \cos \theta \cdot \cos^2 \theta - \sin^2 \theta \cdot \sin \theta)$$

$$= \frac{1}{3} \pi l^3 \cos^2 \theta (2 \cos \theta - \sin^2 \theta)$$

$$\therefore \frac{dV}{d\theta} = 0 \text{ at}$$

$$2\cos^2\theta - \sin^2\theta = 0$$

$$\therefore \tan\theta = \sqrt{2}$$

Further,

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{1}{3}\pi l^3 \sin\theta (\sqrt{2}\cos\theta + \sin\theta) \times (\sqrt{2}\cos\theta - \sin\theta) \\ &= \frac{1}{3}\pi l^3 \sin\theta \cos^2\theta (\sqrt{2} + \tan\theta)(\sqrt{2} - \tan\theta) \\ &= \frac{1}{3}\pi l^3 \sin\theta \cos^2\theta (\tan\theta + \sqrt{2})(\tan\theta - \sqrt{2}) \end{aligned}$$

When α is slightly $< \tan^{-1}\sqrt{2}$

$$\sin\theta \cos^2\theta = +ve, \tan\theta - \sqrt{2} = -ve, \tan\theta + \sqrt{2} = +ve$$

$$\therefore \frac{dV}{d\theta} = (-)(+)(-)(+) = +ve$$

When α is slightly $> \tan^{-1}\sqrt{2}$

$$\sin\theta \cos^2\theta = +ve, \tan\theta - \sqrt{2} = +ve, \tan\theta + \sqrt{2} = +ve$$

$$\frac{dV}{d\theta} = (-)(+)(-)(+) = -ve$$

$\therefore \frac{dV}{d\theta}$ changes sign from +ve to -ve so, V is maximum at $\theta = \tan^{-1}\sqrt{2}$

26. Solution :

The equations of the given curves are

$$x^2 + y^2 = 1 \text{ ----- (i)}$$

and $(x-1)^2 + (y-0)^2 = 1 \text{ ----- (ii)}$

Clearly, $x^2 + y^2 = 1$, represents a circle with centre at $(0, 0)$ and radius units. Similarly 2nd equation represents a circle with centre at $(1, 0)$ and radius unity.

To find the points of intersection of the given curves, we solve (i) and (ii)

Solving (i) and (ii) simultaneously, we find that the two curves intersect at $A(1/2, \sqrt{3}/2)$ and $D(1/2, -\sqrt{3}/2)$.

Since both the curves are symmetrical about x -axis.

So, Required area = 2 (Area $OABCO$)

Now, we slice the area $OABCO$ into vertical strips. We observe that the vertical strips change their character at $A(1/2, -\sqrt{3}/2)$. So,

$$\text{Area } OABCO = \text{Area } OACO + \text{Area } CABCO$$

When area $OACO$ is sliced into vertical strips, we find that each strip has its upper end on the circle $(x-1)^2 + (y-0)^2 = 1$ and the lower end on x -axis. So, the approximating

rectangle shown in Fig. 20.26 has, Length = y_1 , Width = Δx and Area = $y_1 \Delta x$. As it can move from $x = 0$ to $x = 1/2$.

$$\begin{aligned} \therefore \text{Area } CACO &= \int_0^{1/2} y_1 dx \\ &= \int_0^{1/2} \sqrt{1-(x-1)^2} dx \quad \left[\begin{array}{l} \because P(x, y_1) \text{ lies on } (x-1)^2 + y^2 = 1 \\ \therefore (x-1)^2 + y^2 = 1 \Rightarrow y_1 = \sqrt{1-(x-1)^2} \end{array} \right] \end{aligned}$$

Similarly, approximating rectangle in the region $CABC$ has, Length = y_2 , Width = Δx and Area = $y_2 \Delta x$. As it can move from $x = \frac{1}{2}$ to $x = 1$.

$$\begin{aligned} \therefore \text{Area } CACO &= \int_0^{1/2} y_2 dx \\ &= \int_{1/2}^1 \sqrt{1-x^2} dx \quad \left[\begin{array}{l} \because Q(x, y_2) \text{ lies on } x^2 + y^2 = 1 \\ \therefore x^2 + y_2^2 = 1 \Rightarrow y_2 = \sqrt{1-x^2} \end{array} \right] \end{aligned}$$

Hence, required area

$$\begin{aligned} &= 2 \left[\int_0^{1/2} \sqrt{1-(x-1)^2} dx + \int_{1/2}^1 \sqrt{1-x^2} dx \right] \\ &= 2 \left[\left[\frac{1}{2} \cdot (x-1) \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1} \left(\frac{x-1}{1} \right) \right]_0^{1/2} + \left[\frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{1/2}^1 \right] \\ &= \left[\left\{ -\frac{\sqrt{3}}{4} + \sin^{-1} \left(-\frac{1}{2} \right) - \sin^{-1}(-1) \right\} + \left\{ \sin^{-1}(1) - \frac{\sqrt{3}}{4} - \sin^{-1} \left(\frac{1}{2} \right) \right\} \right] \\ &= -\frac{\sqrt{3}}{3} - \frac{\pi}{6} + \frac{\pi}{2} + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{sq. units} \end{aligned}$$

27. **Solution :** The given planes are

$$\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$$

$$\text{and } \vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$$

Taking $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, these equations of the planes become

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$$

$$\text{and } (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow 2x + 6y + 12 = 0$$

and $3x - y + 4z = 0$

Equation of any plane through their intersection is given by

$$(2x + 6y + 12) + \lambda(3x - y + 4z) = 0$$

$$\Rightarrow (2 + 3\lambda)x + (6 - \lambda)y + 4\lambda z + 12 = 0 \quad \dots\dots (1)$$

Now plane (1) is a unit distance from the origin.

$$\therefore \frac{12}{\sqrt{(2 + 3\lambda)^2 + (6 - \lambda)^2 + (4\lambda)^2}} = \pm 1$$

$$\Rightarrow 26\lambda^2 + 40 = 144$$

$$\Rightarrow \lambda^2 = \frac{144 - 40}{26} = \frac{104}{26} = 4$$

$$\Rightarrow \lambda = \pm 2$$

Putting the values of λ in (1), the cartesian equation of planes are

$$8x + 4y + 8z + 12 = 0$$

and $-4x + 8y - 8z + 12 = 0$

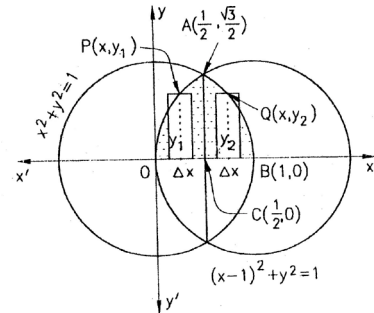
$$\Rightarrow 2x + y + 2z + 3 = 0$$

and $x - 2y + 2z - 3 = 0$

Their vector equations are

$$\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$$

and $\vec{r} \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) - 3 = 0$



28. Solution :

Step-1 : To find the number of bottles of M_1 and M_2 in order to maximise the profit under the given hypotheses.

Step-2 : Let x be the number of bottles of type M_1 medicine and y be the number of bottles of type M_2 medicine. Since profit is Rs 8 per bottle for M_1 and Rs 7 per bottle for M_2 , therefore the objective function (which is to be maximised) is given by $Z = Z(x, y) = 8x + 7y$

The objective function is to be maximised subject to the constraints (Refer Chapter 12 on Linear Programming)

$$\left. \begin{array}{l} x \leq 20000 \\ y \leq 40000 \\ x + y \leq 45000 \\ 3x + y \leq 66000 \\ x \geq 0, y \geq 0 \end{array} \right\}$$

Step-3 : The shaded region OPQRST is the feasible region for the constraints (1)

The co-ordinates of vertices O, P, Q, R, S and T are (0, 0), (20000, 0), (20000, 6000), (10500, 34500), (5000, 40000) and (0, 40000), respectively.

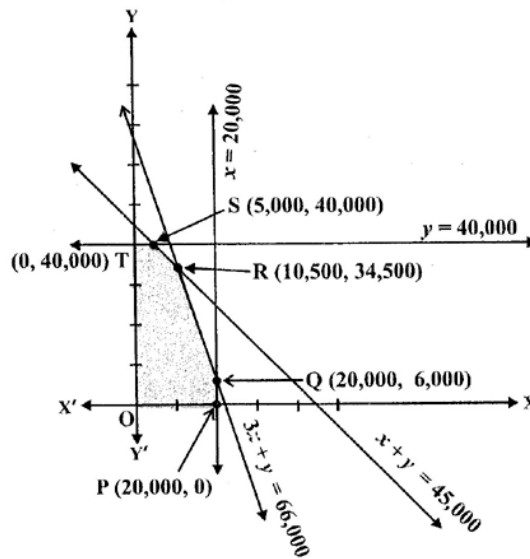


Fig A.2.3

Note that

$$Z \text{ at } P(0, 0) = 0$$

$$Z \text{ at } P(20000, 0) = 8 \times 20000 = 160000$$

$$Z \text{ at } Q(20000, 6000) = 8 \times 10500 + 7 \times 34500 = 325500$$

$$Z \text{ at } R(10500, 34500) = 8 \times 5000 + 7 \times 40000 = 320000$$

$$Z \text{ at } S(5000, 40000) = 8 \times 5000 + 7 \times 40000 = 320000$$

$$Z \text{ at } T(0, 40000) = 7 \times 40000 = 280000$$

Now observe that the profit is maximum at $x = 10500$ and $y = 34500$ and the maximum profit is Rs 325500. Hence, the manufacturer should produce 10500 bottle of M_1 medicine and 34500 bottles of M_2 medicine in order to get maximum profit of Rs 325500.

29. Solution :

Let x denote the number of kings in a draw of two cards. x is a random variable which can assume the values 0, 1 and 2.

$$\begin{aligned} \text{Now, } P(x=0) = P(\text{no king}) &= \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{\frac{48!}{2!(48-2)!}}{\frac{52!}{2!(52-2)!}} \\ &= \frac{48 \cdot 47}{52 \cdot 51} = \frac{188}{221} \end{aligned}$$

$$P(x = 1) = P(\text{one king or one non king})$$

$$= \frac{{}^4C_2 {}^4C_1}{{}^{52}C_2}$$

$$= \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}$$

and $P(x = 2) = P(\text{two kings}) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51}$

$$= \frac{1}{221}$$

Thus probability of distribution of x is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Now, Mean of $X = E(X) = \sum_{i=1}^n x_i P(x_i)$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$

$$= \frac{34}{221}$$

Also, $E(X^2) = \sum_{i=1}^n x_i^2 P(x_i)$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$$

$$= \frac{36}{221}$$

Now, $Var(X) = E(X^2) - [E(X)]^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$

$$= \frac{6800}{(221)^2}$$

$$\therefore \sigma = \sqrt{\text{var}(X)} = \frac{\sqrt{6800}}{221} = .37$$

MODEL TEST PAPER-3

General Instruction As in Set-1

1. If f be a function defined by $f(x) = \frac{ax-b}{bx-a}$, find $f(f(x))$.
2. Find the value of x such that $\cot^{-1} x = \tan^{-1} \frac{1}{x}$ holds good.
3. If A and B are symmetric matrices of the same order, then prove that $AB + BA$ is symmetric.
4. If $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = [1 \ 5 \ 7]$, find $B'A'$.
5. Find the values of k if the area of triangle formed by the vertices $(k, 0)$, $(4, 0)$, $(0, 2)$ is 4 square units.
6. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$
7. Evaluate $\int xe^x dx$
8. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.
9. If Q be the angle between \vec{a} and \vec{b} , then for which value of θ , $\vec{a} \cdot \vec{b} \geq 0$?
10. Find the Cartesian equation of the line that passes through the origin and $(5, -2, 3)$.
11. If A be the set of all numbers except 1 and O be an operation on A defined by $aob = a + b - ab$ for all $a, b \in A$, prove that "O" is associative.
12. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x . Show that

$$4 \tan^{-1} \left(\frac{1}{5} \right) - \tan^{-1} \frac{1}{70} + \tan^{-1} \left(\frac{1}{99} \right) = \frac{\pi}{4}$$

13. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$
14. Find the relationship between a & b so that the function f defined by $f(x) = \begin{cases} ax+1, & x \leq 3 \\ bx+3 & x > 3 \end{cases}$ is continuous at $x = 3$.
15. Find $\frac{dy}{dx}$ when $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$, $0 < x <$

OR

Find $\frac{dy}{dx}$ if $y = (\sin x)^x + \sin^{-1} \sqrt{x}$

16. Evaluate $\int (\sqrt{\cot x} + \sqrt{\tan x}) dx$

OR

Integrate $I = \int \sqrt{1-4x-x^2}$

17. Using properties of definite integrals, evaluate $\int_0^{\pi/4} \log(1+\tan x) dx$

OR

Find $\int_0^2 (x^2+1) dx$ as the limit of a sum.

18. Find the general solution of the differential equation $(e^2 + e^{-x}) dy - (e^x - e^{-x}) dx = 0$.

19. Find the particular solution of the differential equation $\frac{dx}{dy} + y \cot x = 2x + x^2 \cot x$ ($x \neq 0$),

given that $y = 0$ when $x = \pi/2$

20. If \vec{a} , \vec{b} , \vec{c} are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

21. Find the angle between the planes whose equations are $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$

22. A card from a pack of 52 cards is lost from the remaining cards of the pack 2 cards are drawn and are found to be both diamonds. Find the probability that the lost card was a diamond.

23. Using elementary transformation, Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

OR

Using properties of determinants, show that

$$\begin{bmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{bmatrix} = abc \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1 \right)$$

24. Show that the semi vertical angle of the cone of the maximum volume of given slant height is $\tan^{-1} \sqrt{2}$.

OR

Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

25. Using integration find the area of the region bounded by the triangle whose vertices are $(3, 0)$, $(2, 2)$ and $(3, 1)$.

OR

Find the area of the region enclosed between the two curves given by $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$

26. Show that the differential equation $ydx + x \log \frac{y}{x} dy - 2xdy = 0$ is homogenous and then solve it.
27. Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

28. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000/- on each piece of model A and Rs. 12000/- on each piece of model B. How many pieces of model A and model B should be manufactured per week to realise a maximum profit. What is the maximum profit per week?
29. Find the variance of the number obtained on a throw of an unbiased die.

ANSWERS SET-3

1. x

2. $x > 0$

4. $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

5. 0, 8

6. $\tan \frac{\theta}{2}$

7. $(x-1)e^x + C$

8. $\pm \frac{1}{\sqrt{3}}$

9. $0 \leq \theta \leq \pi/2$

10. $\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$

12. $\pm \frac{1}{\sqrt{2}}$

14. $a = b + \frac{2}{3}$

15. $\frac{2}{1+x^2}; (\sin x)^2 (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$

16. $\sqrt{2} \tan^{-1}\left(\frac{\tan \theta - 1}{\sqrt{2} \tan \theta}\right) + C; \quad \frac{5}{2} \sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + \frac{x+2}{2} \sqrt{1-4x-x^2} + C$

17. $\frac{\pi}{8} \log 2; \quad \frac{14}{3}$

18. $y = \log|e^x + e^{-x}| + C, \quad x \in R$

19. $y = x^2 - \frac{\pi^2}{4 \sin x} (\sin x \neq 0)$

20. $\frac{-3}{2}$

21. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$

22. $\frac{11}{50}$

23. $\begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$

24. $x+14y-254=0, \quad x+14y+86=0$

25. $\frac{3}{2}; \frac{8\pi}{3} - 2\sqrt{3}$

26. $cy = \log \frac{y}{x} - 1$

27. $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

28. 12 pieces of Model A and 6 pieces of Model B should be manufactured to realise maximum profit of Rs. 1,68,000.

29. $\frac{35}{12}$

MODEL TEST PAPER-4

General Instruction as in Set-1

1. Give an example of a function which is neither one one nor onto.
2. Evaluate $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$
3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then for all $n \in N$, what is A^n ?
4. If a matrix has 24 elements, what are the possible nos of orders it can have?
5. Find $\text{adj } A$ for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$
6. If $y = \sin(\tan^{-1} e^x)$, then find $\frac{dy}{dx}$
7. Find the value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$
8. If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, find $|\vec{x}|$
9. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{b} = \hat{i} - \hat{j} + \hat{k}$
10. Find the distance the planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$
11. Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is bijective.
12. Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0,1]$

OR

Show that $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4}\right)$

13. If a,b,c are positive & unequal, show that the value of the determinant\

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ is negative.}$$

14. Show that the function $f(x) = |\cos x|$ is a continuous function

15. Find $\frac{dy}{dx}$, if $y = (\log x)^x + x^{\log x}$

OR

Verify mean value theorem for $f(x) = x^3 - 5x^2 - 3$ is $[1,3]$

16. Integrate $\int \log |4x^2 - 1| dx$

OR

Integrate $\int \frac{dx}{e^x - 1}$

17. Evaluate $\int_0^{\pi/2} (2\log \sin x - \log \sin 2x) dx$

OR

Evaluate $\int_0^1 |2x - 1| dx$

18. Solve the differential equation

$$\frac{dy}{dx} - \tan x \cdot y = \sin x e^{\sin x}, \quad 0 < x < \frac{\pi}{2}$$

19. Form the differential equation of the family of hyperbolas having foci on x -axis and centre at origin.

20. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} , and $\vec{c} \cdot \vec{d} = 15$

21. Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$$

22. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the

probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A.

23. The sum of three number is 6. If third number is multiplied by 3 and added to second number the result is 11. By adding first and third numbers, double of the second no is obtained. Represent it algebraically and find the numbers using matrix.

OR

If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, find A^{-1} using elementary transformation.

24. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

OR

Find the interval in which the function $f(x) = 10 - 6x - 2x^2$ is strictly decreasing or increasing.

25. Find the arca of the region

$$\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$$

OR

Prove that the curves $y = x^2$ and $x = y^2$ divide the square bounded by $x = 0, y = 0, x = 1, y = 1$ into three equal parts.

26. Solve the differential equation

$$x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}, \quad x > 0$$

27. Find the image of the line $\frac{x-1}{0} = \frac{y-3}{1} = \frac{z-4}{7}$ in the plane $2x - y + z + 3 = 0$

28. A merchant plans to sell two types of personal computers a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computer will not exceed 250 units. Determine the number of units of each type of computers which the merchant should store to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

29. If a fair coin is tossed 10 times, find the probability of getting

- (i) Exactly six heads.
- (ii) At least six heads.
- (iii) At most six heads.

ANSWERS (SET-4)

2. $\frac{\pi}{6}$

3. $\begin{bmatrix} n & 0 \\ 1 & 1 \end{bmatrix}$

4. 8

5. $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

6. $\frac{e^x \cos(\tan^{-1} e^x)}{1+e^{2x}}$

7. $\frac{\pi}{12}$

8. 3

9. $\cos^{-1}\left(-\frac{1}{3}\right)$

10. $\frac{2}{\sqrt{29}}$ units

15. $(\log x)^{x-1} [1 + \log x \cdot \log(\log x)] + 2x^{\log^{-1}} \cdot \log x$

16. $x \log|4x^2 - 1| - 2x - \log\left|\frac{2x-1}{2x+1}\right| + C ; \log\left(\frac{e^x - 1}{e^x}\right) + c$

17. $\frac{\pi}{2} \log \frac{1}{2}; \frac{1}{2}$

19. $xyy'' + x(y')^2 - yy' = 0$

20. $\frac{1}{3}(160\hat{i} - 5\hat{j} + 70\hat{k})$

21. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

22. $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ or $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

23. $x = 1, y = 2, z = 3$; $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

24. 25, 10; strictly decreasing for $x > -\frac{3}{2}$ & strictly increasing for $x < -\frac{3}{2}$

25. $\frac{23}{6}$

26. $y + \sqrt{x^2 + y^2} = Cx^2$

$$27. \frac{x+3}{0} = \frac{y-5}{1} = \frac{z-2}{7}$$

Image of (1, 3, 4) is (-3, 5, 2); required image will be the line passing through (-3, 5, 2) and parallel to the given line.

28. 200 units desktop model and 50 unit of portable model. Maximum profit Rs. 1150000.

$$29. \text{(i) } \frac{105}{512} \quad \text{(ii) } \frac{193}{512} \quad \text{(iii) } \frac{53}{64}$$

MODEL TEST PAPER-5

Sub. : Maths

Max. Marks : 100

Class : XII

Time : 3 hrs.

General Instruction as in Set-1

SECTION-A

- Find the principal value of $\sin^{-1}\left(\sin 4\frac{\pi}{3}\right)$
- A matrix has 6 elements. Write the possible orders of the matrix.
- Evaluate $\begin{vmatrix} \sin x & \cos x \\ \cos x & \sin x \end{vmatrix}$ when $x = \frac{\pi}{4}$
- Find the abscissa for which the tangent to the curve $y = x^2 - 5$ is parallel to the line $y = -x + 6$.
- Find the angle between \vec{a} and \vec{b} such that $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{3}$ and $|\vec{a} \times \vec{b}| = 3$.
- Find the direction ratio's of the line $6x - 1 = 2y + 3 = 5 - z$.
- Evaluate $\int e^{5 \log x} \sqrt{x} dx$.
- Find $15 * (-3)$ if $a * b = 2a - b$ $a, b \in \mathbb{Z}$.
- If A and B are symmetric matrices of the same order, under what condition you can say that AB is also symmetric.
- Find λ such that the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ are orthogonal.

SECTION-B

- Show that the relation R in real numbers defined as $R = \{(a, b) : a < b\}$ is transitive but neither reflexive nor symmetric.

OR

Let $f : N \rightarrow R$ be a function defines as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow \text{Range of } f$ is invertible. Also find the inverse of f .

12. Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

13. If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$ find $(AB)^{-1}$

OR

Using the properties of determinants prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

14. Examine the continuity of the function $f(x) = |x - 5|$.

15. Differentiate $\log_7(\log x)$ w.r.t. x .

OR

Differentiate $\sin^2 x$ with respect to $e^{\cos x}$

16. Evaluate $\int \frac{5dx}{1 + \tan x}$

17. For what value of x the function $f(x) = \frac{x}{2} + \frac{2}{x}$ ($x \neq 0$) is increasing or decreasing on R .

18. Evaluate $\int_a^b x^2 dx$ as limit of sum.

19. Show that $\int_0^{\pi/4} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$.

20. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$ such that each is perpendicular to the sum of other two find $|\vec{a} + \vec{b} + \vec{c}|$.

21. Find the coordinates of the point where the line through A (3, 4, 1) and B (5, 1, 6) crosses xy -plane.

22. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and the variance of the number of kings.

OR

A pair of dice is thrown 7 times. If getting total of 7 is considered a success. What is the probability of at least 6 successes.

SECTION-C

23. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1} using A^{-1} solve the following system of linear equations

$$2x - 3y + 5z = 11, \quad 3x + 2y - 4z = -5, \quad x + y - 2z = -3$$

24. Find the local maximum and local minimum values if any for the function:

$$y = (x-3)^3 (x-4)^4$$

OR

A cylinder is such that the sum of its height and circumference of its base is 10 metres. Find the maximum volume of the cylinder.

25. Draw the rough sketch of the region $\{(x, y) : y^2 \leq 4x, \quad 4x^2 + 4y^2 \leq 9\}$ and find enclosed area using integration.

26. Solve the differential equation $y - x \frac{dy}{dx} = a \left(y^2 + x^2 \frac{dy}{dx} \right)$ $y(a) = a$

27. A farmer decides to plant upto 10 hectares with cabbage and potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbage and at least one but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1500 per hectare on cabbage and Rs. 2000 per hectare on potatoes, how should he plan his farming so as to get the maximum profit, assuming the total yield that he gets is sold.

28. Find the equation of the plane through the intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 1 = 0$.

29. Three urns contain 3 green and 2 white balls, 5 green and 6 white balls, 2 green and 4 white balls respectively. One ball is drawn from each urn. Find the mean various of the probability distribution of the random variable "Number of white balls drawn".

OR

The probability that a bulb produced in a factory will fuse after 150 days of use is 0.05. Find the probability that out of such bulbs (i) one (ii) not more than one (iii) more than one (iv) at least one will fuse after 150 days of use.

ANSWERS

1. x
2. $x > 0$

$$4. \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$5. 0, 8$$

$$6. \tan \frac{\theta}{2}$$

$$7. (x-1)e^x + C$$

$$8. \pm \frac{1}{\sqrt{3}}$$

$$9. 0 \leq \theta \leq \frac{\pi}{2}$$

$$10. \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

$$12. \pm \frac{1}{\sqrt{2}}$$

$$14. a = b + \frac{2}{3}$$

$$15. \frac{2}{1+x^2}; (\sin x)^2 (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$$

$$16. \sqrt{2} \tan^{-1} \left(\frac{\tan \theta - 1}{\sqrt{2} \tan \theta} \right) + C; \quad \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + \frac{x+2}{2} \sqrt{1-4x-x^2} + C$$

$$17. \frac{\pi}{8} \log 2; \quad \frac{14}{3}$$

$$18. y = \log |e^x + e^{-x}| + C, \quad x \in R$$

$$19. y = x^2 - \frac{\pi^2}{4 \sin x} (\sin x \neq 0)$$

$$20. \frac{-3}{2}$$

$$21. \cos^{-1} \left(\frac{15}{\sqrt{731}} \right)$$

$$22. \frac{11}{50}$$

$$23. \begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$24. x+14y-254=0, \quad x+14y+86=0$$

$$25. \frac{3}{2}; \quad \frac{8\pi}{3}-2\sqrt{3}$$

$$26. cy = \log \frac{y}{x} - 1$$

$$27. \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

28. 12 pieces of Model A and 6 pieces of Model B should be manufactured to realise maximum profit of Rs. 1,68,000.

$$29. \frac{35}{12}$$

MODEL TEST PAPER-6

Class - XII

Sub : Mathematics

Time : 3hrs

M.M. : 100

General Instruction : As in set 1

01. If a binary operation ‘*’ is defined by $a * b = a^2 + b^2 + ab + 1$, $\tan+1$ then find $(2 * 3) * 2$

02. Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) + \operatorname{cosec}^{-1} \frac{2}{\sqrt{3}}$

03. Construct a 2×2 Matrix. Whase element a_{ij} are given by $a_{ij} = i + j$

04. Let $A = \begin{bmatrix} a & o & o \\ o & a & o \\ o & o & a \end{bmatrix}$ find A^n

05. Using the property of determinant and without expending prove

$$\begin{bmatrix} \frac{1}{a} & a & bc \\ \frac{1}{b} & b & ca \\ \frac{1}{c} & c & ab \end{bmatrix} = 0$$

06. Evaluate : $\int_0^{\pi/2} \sin^2 x \, dx$

07. Evaluate : $\int_1^e \frac{\log x}{x} \, dx$

08. Find the direction cosine of a line which makes equal angle with coordinate axis

09. If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$ find $\vec{a} \cdot \vec{b}$

10. If $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector then find the value of x

SECTION - B

11. Consider $f : \mathbb{R}_+ \rightarrow [4, \infty]$ given by $f(x) = x^2 + 4$. Show that f is invertible. Also find f^{-1}

12. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ prove that $x^2 + y^2 + z^2 + 2xyz = 1$

OR

Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{62}{16} = \pi$

13. Find all the points of continuity of f defined by $f(x) = |x| - |x+1|$

14. If $x = a(1 + \cos \theta)$, $y = a(\theta + \sin \theta)$ prove that $\frac{d^2 y}{dx^2} = \frac{-1}{a}$ at $\theta = \frac{\pi}{2}$

OR

If $\log y = \tan^{-1} x$ show that $(1+x^2)y_2 + (2x-1)y_1 = 0$

15. Find the equation of tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cut the x -axis.

OR

Let I be any interval disjoint from $(-1, 1)$ prove that the function f is given by $f(x) = x + \frac{1}{x}$ is increasing on I .

16. Using the property of definite integral. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} \, dx$

OR

Evaluate $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

17. Show that the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

18. Solve $\frac{dy}{dx} - y = \cos x$

19. Find the equation of plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

20. If \hat{a} and \hat{b} are unit vector inclined at angle θ then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$.

21. A man speak truth 4 out of 5 times. He throws a dice and reports that it is six. What is probability that it is actually six.

22. If a, b, c are all positive and are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P. then show that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

23. Solve the following equation by matrix method –

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

OR

Find the inverse using elementary row transformation –

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

24. Show that height of cylinder of greatest volume which can be inscribed in a right circular cone of height h and semivertical angle α is one third of height of that cone and greatest

volume of cylinder is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.

OR

The sum of the perimeter of a circle and square is K where K is some constant. Prove that the sum of their areas is least when the side of square is double the radius of circle.

25. Evaluate as limit of sum $\int_0^4 (x + e^{2x}) dx$

26. Find the area of the region bounded between $y = x^2$ and $y = |x|$.
27. Find the shortest distance between the lines $\vec{r} = (3\hat{i} + \theta\hat{j} + \theta\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (-3\hat{i} + 7\hat{j} + 6\hat{k}) + \mu(-3\hat{i} - 2\hat{j} + 4\hat{k})$. Also find the equation of S.D.
28. Two cards are drawn simultaneously without replacement from a pack of 52 cards. Find mean, variance and S.D. for the number of kings.
29. A man rides his motor cycle at 25 km/hr. He has to spend Rs. 4 per km on petrol. If he rides at a faster speed of 40 km/hr. the petrol cost increase to Rs. 10 per km. He has Rs. 200 to spend on petrol and wishes to find maximum distance. He can travel within one hour. Express this as a L.P.P. then solve it.

ANSWERS

1. 445
2. 0
3. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$
4. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$
6. $\frac{\pi}{4}$
7. $\frac{1}{2}$
8. $\pm \frac{1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}$
9. ± 7
10. $\pm \frac{1}{\sqrt{3}}$
13. No point of discontinuity.
16. $\frac{\pi^2}{4}$ OR $e^x \tan \frac{x}{2} + C$
17. $\sin\left(\frac{y}{x}\right) = \log|cx|$
18. $y = \left(\frac{\sin x - \cos x}{2}\right) + ce^x$

19. $33x + 45y + 50z - 41 = 0$

21. $\frac{4}{9}$

23. $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

OR

$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

25. $\frac{15 + e^\theta}{2}$

26. $\frac{1}{3}$

27. S.D. = $3\sqrt{30}$, $\vec{r} = (3\hat{i} + \theta\hat{j} + 3\hat{k}) + \mu(-6\hat{i} - 15\hat{j} + 3\hat{k})$

28. $\frac{34}{321}, \frac{6800}{(221)^2}, 0.37$

29. Maximum distance = 30 km.

MODEL PRACTICE PAPER-7

Class - XII

Sub : Mathematics

Time : 3hrs

M.M. : 100

General Instruction :

- All questions are compulsory. However, internal choices have been provided.
- Section A consist of 10 questions of one marks each. Section B consist of 12 questions of four marks each. Section C consist of 7 question of six marks each.

- If the binary operation ‘*’ on z defined by $a * b = a^2 - b^2 + ab + 4$ then find the value of $(2 * 3) * 4$.

- Evaluate $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

- For what value of x the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular.

4. Using determinant prove that $\frac{1}{a} + \frac{1}{b} = 1$ if the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear.
5. If $A = \text{diag}(2, -5, 9)$, and $C = \text{diag}(-6, 3, 4)$ find $A - B - C$.
6. Evaluate $\int \sin^2 \frac{x}{2} dx$
7. Evaluate $\int \sec^3 x \tan x dx$
8. If a line makes 30° , 60° and 90° with positive direction of x , y and z -axis find its direction cosine.
9. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ show that \vec{a} and \vec{b} are perpendicular.
10. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ show that $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$.

SECTION-B

11. Show that the function $f : R - \{3\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is invertible. Also find f^{-1} .
12. If $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$

OR

Prove that $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right\} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$

13. Discuss the continuity of the function given by $f(x) = |x-1| + |x-2|$ at $x=1$ and $x=2$.
14. If $y = (\tan^{-1} x)^2$ then prove that $(1+x^2)^2 y_2 + 2x(1-x^2) y_1 = z$

OR

If $x^m y^n = (x+y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$.

15. Prove that the curve $x = y^2$ and $xy = k$ cut at right angle if $\theta k^2 = 1$.

OR

Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$.

16. Using the property of definite integral. Evaluate $\int_0^\pi \frac{x dx}{1 + \sin x}$

OR

Evaluate: $\int (\sin^{-1} x)^2 dx$

17. Solve : $x \frac{dy}{dx} + 2y = x^2$

18. Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2} dx$ is homogeneous and solve it.

19. Find the equation of plane passing through the lines of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

20. If \vec{a} is any vector in space show that $\vec{a} = (\hat{a} \cdot \hat{i})\hat{i} + (\hat{a} \cdot \hat{j})\hat{j} + (\hat{a} \cdot \hat{k})\hat{k}$

21. Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls one ball is drawn at random from one of the bags and it is found to be red. find the probability that it was drawn from bag II.

22. Prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

SECTION-C

23. The sum of three numbers is 6. If we multiply the third by 2 and add the first number to the result, we get 7 by adding second and third numbers to three times the first number, we get 12. Using matrix find the number.

OR

Find the inverse using elementary row transformation

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

24. Show that semi vertical angle of right circular cone of given surface and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

OR

Prove that volume of largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.

25. Evaluate as limit of sums $\int_0^2 (x^2 + 2x + 1) dx$

26. Find the area of the region bounded between $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$.
27. Show that the lines $\frac{x-1}{2} = \frac{y-3}{4} = -z$ and $\frac{x-4}{3} = \frac{1-y}{2} = z-1$ are coplanar. Also find the equation of plane containing them.
28. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice. Also find the mean and variance.
29. A toy company manufactures two type of dolls A and B market test and available resource have indicated that the combined production level should not exceed 1200 doles per week and demand of dolls of type B is at most half of that for dolls of type A. Further three times the production of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on doll A and B. How many of each should be produced weekly in order to maximise the profit.

ANSWERS

1. 33
2. $\frac{3\pi}{4}$
3. -1
5. $\text{diag}(7, -9, 9)$
6. $\frac{1}{2}(x - \sin x) + C$
7. $\frac{\sec^3 x}{3} + C$
8. $\frac{\sqrt{3}}{2}, \frac{1}{2}, 0$
13. Continious at $x = 1$ and $x = 2$.
15. Increasing in $(0, \pi/4)$ and $(\frac{5\pi}{4}, 2\pi)$
Decreasing in $(\pi/4, 5\pi/4)$
16. p OR $(\sin^{-1} x)^2 \cdot x + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$
17. $y = \frac{x^2}{4} - cx^{-2}$
18. $y + \sqrt{x^2 + y^2} = cx^2$
19. $y - 3z + 6 = 0$

21. $\frac{35}{68}$

22. $x = 3, y = 1, z = 2$ OR

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$

25. $\frac{26}{3}$

26. $(\pi - 2)\frac{ab}{4}$

27. $2x - 5y - 16z + 13 = 0$

28. $\frac{9}{16}, \frac{6}{16}, \frac{1}{16}$ and mean = $\frac{3}{2}$, Var = $\frac{3}{8}$

29. Type A \rightarrow 800, Type B \rightarrow 400, Max. Profit = Rs. 16000.

MODEL PRACTICE PAPER-8

General Instruction : As in set 1

SECTION-A

1. If $f : R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$ find $f[f(x)]$.
2. Using principal value evaluate the following $\cos^{-1}\left(\cos^2 \frac{\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$
3. Using determinant if $(2, -3), (\lambda, -1)$ and $(0, 4)$ are collinear find the value of λ .
4. Evaluate $\begin{vmatrix} \tan 30^\circ & \cot 30^\circ \\ -\tan 60^\circ & \cot 60^\circ \end{vmatrix}$
5. Let A be non singular square matrix of order 3×3 . Then $|adjA|$ is equal to?
6. Evaluate $\int \frac{1 - \sin x}{\cos^2 x} dx$
7. Evaluate $\int_2^3 \frac{xdx}{1+x^2}$

8. If $\vec{p}(1,5,4)$ and $\vec{q}(4,5,-2)$ find the direction ratios of \overline{pq} .
9. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.
10. If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$ then find $|\vec{x}|$.

SECTION-B

11. Let N be the set of all natural numbers and let R be a relation on $N \times N$ defined by $(a,b)R(c,d) \Leftrightarrow ad = bc$ for all $(a,b), (c,d) \in N \times N$ show that R is an equivalence relation.

12. Prove that $\tan^{-1} \left\{ \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right\} = \frac{\pi}{4} + \frac{x}{2}, 0 < x < \frac{\pi}{2}$

OR

Solve for x , $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$

13. Let $f(x) = \begin{cases} \frac{1-\cos x}{x^2} & ; x < 0 \\ \sqrt{x} & x = 0 \\ \sqrt{16+\sqrt{x}} - 4 & x > 0 \end{cases}$, determine the value of a if possible so that the function

is continuous at $x = 0$.

14. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ show that $\frac{dy}{dx} = \frac{-y}{x}$.

OR

Find $\frac{dy}{dx}$ if $y = \tan^{-1} \left[\frac{\sqrt{1+a^2x^2} - 1}{ax} \right]$

15. Find the intervals in which $f(x) = 2x^3 - 9x^2 + 12x + 15$ is (i) increasing and (ii) decreasing.

OR

Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$.

16. Evaluate: $\int \frac{1}{\sin x(2+\cos x)} dx$

OR

$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

17. Solve $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$

18. Solve $x^2 \frac{dy}{dx} = y^2 + 2xy$ given that $y = 1$ when $x = 1$.

19. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vector $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ .

20. Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ also find the length of the perpendicular.

21. A dice is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once.

22. Show that :

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+2 & z+x & x+y \end{vmatrix} = z \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

SECTION-C

23. Using matrices solve the following system of linear equation $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$.

OR

Using elementary transformations find the inverse of the following matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

24. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

A window is in the form of a rectangle surrounded by a semi circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

25. Evaluate : $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

26. Find the area lying above the x -axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4ax$.
27. Find the equation of the plane through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$.
28. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver and a truck is 0.01, 0.03 and 0.015 respectively one of the insured person meets with an accident. What is the probability that he is a scooter driver?
29. A dietician wishes to mix two types of food is such away that the vitamin content of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs.5.00 per kg to purchase food 1 and Rs. 7.00 per kg to produce to food II. Determine the minimum cost to such a mixture formulate L.P.P. and solve it.

ANSWERS

1. $x^4 - 6x^3 + 10x^2 - 3x$
2. λ
3. $\frac{10}{7}$
4. $\frac{10}{3}$
5. $|A|^2$
6. $\tan x - \sec x + C$
7. $\frac{1}{2} \log \frac{3}{2}$
8. $\left(\frac{3}{7}, \frac{-6}{7}, \frac{-2}{7} \right)$
9. $\frac{60}{\sqrt{114}}$
10. 3
12. $\frac{\pi}{4} + \frac{x}{2}$ or $\pm \frac{1}{\sqrt{2}}$
13. $a = 8$
15. $f(x)$ is increasing in $\{x : x > 2\} \cup \{x : x < 1\}$
 $f(x)$ is decreasing in $\{x : 1 < x < 2\}$

OR

$$(2, -2); \left(-\frac{2}{3}, -\frac{14}{27}\right)$$

$$16. \frac{1}{6} \log|1 - \cos x| - \frac{1}{2} \log|1 + \cos x| + \frac{1}{3} \log|2 + \cos x|$$

$$17. y(x^2 + 1)^2$$

$$18. y^2 = \frac{x^2}{2 - x}$$

$$19. \lambda = 1$$

$$20. (2, 3, -1); \sqrt{21}$$

$$21. \frac{2}{5}$$

$$23. x = 3, y = 4, z = 1$$

$$24. \text{Height} = \frac{10}{\pi + 4} \text{ cm}, \text{ Breadth} = \frac{20}{\pi + 4} \text{ m}$$

OR

$$\text{Max. volume} = \frac{8}{27} \left(\frac{4}{3} \pi R^3 \right)$$

$$25. a \left(\frac{\pi}{2} - 1 \right)$$

$$26. \frac{4}{3} (8 + 3\pi)$$

$$27. 18x + 17y + 4z - 49 = 0$$

$$28. \frac{1}{52}$$

$$29. \text{Minimum Value} = \text{Rs. } 38.00$$